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BAYESIAN REGULARIZATION IN THE THEORY OF STAP

After adding some details to Melvin's overview on theory and technique of STAP, we point out the role of accounting for Bayesian methods in adaptation theory. In this regard, we develop the relationship between STAP and CFAR shown by E. Kelly but from Bayesian positions and propose the new technique for co-variance matrix estimate regularization optimized from the Bayesian positions. The Bayesian regularization synthesized in the paper is tested using computer simulation.

bayesian regularization, theory of STAP, computer simulation

Introduction

Detailed characteristic of STAP as a branch of science and review of works made in it are presented in the overview by W. Melwin [1]. Since there were no Russian-language works mentioned there, we give, just in case, some information on them. The Space-Time Adaptive Processing (STAP) is a part of Space-Time Processing (STP). The first Russian-language statistical work [2] on STP accounting for resolution peculiarities of several signals was published in 1961 on the basis of similar work [3] of 1959 about the time processing (TP) accounting for resolution peculiarities too. In 1963-64, the first (apparently) typical STAP experiments were made near Kharkov [4] on the basis of [2, 5, 6]. Using the simplest STAP device – multichannel correlation quadrature auto canceller [6] it was possible to clear out the plan-position indicator and observe targets, i.e. the targets became observable – resolved (discriminated) from the interference source.

The problems of polarization and various combined types of selection (angle-polarization, velocity, angle-velocity etc.) were simultaneously considered.

The publications [7], 1965, and [8], 1967, have shown that first works on STAP in USA were also made in the direction of providing signal discrimination from interference using the simplest analog correlation side lobe cancellers operating without direct estimation of covariance matrix (CM) of interference. For the case of TP accounting not peculiarities of resolution, the advisability of such estimation followed from basic statistical works by D. Middleton (USA) [9], 1957, and L. Vainshtein (USSR) [10], 1960, from the works on STP and TP accounting for the peculiarities of resolution [2, 3], as well as from the experiments on STAP of 1963-64 mentioned above. In these experiments, three discrete channels of space-time processing were implemented in hardware. Digital estimation of CM hadn't been implementable at that time due to limited possibilities of digital components.

Among the theoretical works on direct estimation of CM with regard to STP referenced in [1], the first in this field work [11] of 1974, where for the first time the advantages of CM estimation were shown, and, especially, the works [12, 13] of 1986-1992 should be mentioned. Authors of [12, 13] successfully related the STAP devices with the CFAR technique having shown their statistical equivalence. Unfortunately, the works [11 – 13] were based on the maximum likelihood (ML) estimate of interference CM, and not on the Bayesian one. It is well known that Bayesian approach proved to be very effective in the theory of Kalman filtering. Nevertheless, the refuse of using a priory data in adaptation allowed obtainment of many interesting results. However, according to detection curves given in [12, 13], the number of needed snapshot vectors ν have to be larger than the dimensionality of CM m. In the early experiments, emissions of single interference source were significantly easier and not harder to cancel than those of two sources. In other words, the ML method without regularization was doomed to certain slowing down the convergence of $m \times m$ CM estimate (including scalar case m=1, CFAR).

Some earlier than [12, 13] were published, heuristic method of significant improving the convergence of ML estimate of interference CM [14] – method of CM estimate regularization (diagonal loading) – was proposed for the first time. The aforesaid regularization is implemented by adding weighted unit matrix to the ML estimate of CM. The work [15] 2001 develops [14] with regard to specific applications to measurement, with regard to differentiation in eigenvalues weighting and using for this purpose special methods of mathematical programming, etc. Well before [14, 15], the iterative procedures for inverting varying in time CM estimate were developed, in which the process started from the diagonal CM of receiver's noise [16, 17]. Since the necessity of Bayesian approach wasn't mentioned in the works [14 – 17], it is interesting to investigate various Bayesian approaches in order to ground or optimize heuristic methods of CM estimate regularization.

When regularization of CM estimate is absent, the problem arises [18], 2001 of switching the adaptation devices off if the correlation in the input realization drops lower some predetermined level. Let's note, just in case, that this can also be accounted for within Bayesian framework.

The purpose of this paper is to present and generalize the Bayesian theory of Pareto-optimal CFAR and STAP devices [19 – 22]. It is shown that regularization technique of CM estimate follows directly from a priory data. Such datum is, for instance, the easily obtainable level of receiver's (system's) thermal noise. To simplify the analysis, we consider the scalar case first of single channel reception (CFAR), and then we show how can we account for a priory information while estimating the CM. In the multi-channel case we replace the Bayesian estimate of detection probability with corresponding CM estimate, i.e. we deviate from the method presented for the scalar case. Therefore, we refer to this method as the simplified method of Bayesian regularization.

The both scalar and multi channel reception cases are considered in assumption of presence of separate sets of data for estimating the interference parameters and signal detection on interference background. The interference from signal detection channel is not used for estimating the interference parameters. In other words, the use of General Likelihood Ratio Test (GLRT) [12] is not presumed, which is insufficiently effective according to [13].

Using computer simulation we compare quantitatively the simplified variant of Bayesian regularization of CM estimate with other known estimates by their convergence to maximum signal-to-interference ratio depending on number of snapshot vectors ν in case of STAP, and give some recommendations as to refuse from pure ML methods without regularization.

A priory data with regard to single channel reception

Designing CFAR detectors, we may consider two different kinds of initial data. The first kind of data is the receiver's noise level when external interference is absent. The second kind of data is the distribution model (probability density) of external interference variance (uniform, diminishing, and so on). We suppose that not only inner but also external interference has the form of the stationary Gaussian noise. These data can be accounted for by Pareto distribution

$$p(D)=D^{\eta}\Big/\int_{D_0}^{A}D^{\eta}\,dD\,. \qquad (1)$$

Here, D_0 is the receiver's own noise variance; A is the upper bound of the combined interference variance; η is the parameter of variance distribution (if $\eta = 0$ the distribution is uniform, and for $\eta = -2$ only the Bayesian solution transforms into ML one).

Adaptive CFAR detection using Bayesian statistics

To implement any CFAR detector statistically, one must find equation for setting the threshold level given the conditional false alarm probability F. We start the analysis from equation relating F with threshold level for the case of the fixed (but unknown) interference variance D

$$F(Z_0\mid D)=\exp\left(-Z_0^2/2D\right)=\exp\left(-\varsigma_0\right), \qquad (2)$$

where Z0 is the absolute and $\varsigma_0=Z_0^2/2D$ relative (power) threshold level. But in our case the variance D isn't known a priory but it conditions the statistics $s=s(\mathbf{Y})=\sum_{i=1}^{\nu}\left|Y_i\right|^2$ of training sample $Y_1,Y_2,...Y_\nu$. The random value s evaluated after training has the chi-square distribution

$$p(s\mid D)=\frac{s^{(\nu-2)/2}}{\Gamma(\nu/2)(2D)^{\nu/2}}\exp(-s/2D)\,. \qquad (3)$$

Adaptive threshold level can be found after averaging (2) over all possible values of D

$$F(Z_0\mid s)=\frac{\int_{D_0}^{A}F(Z_0\mid D)p(s\mid D)p(D)\,dD}{\int_{D_0}^{A}p(s\mid D)p(D)\,dD}\,, \qquad (4)$$

which reduces to the transcendent equation for relative threshold level ς_0 for given F

$$\frac{\gamma\left(\frac{s}{2D_0}+\varsigma_0,k\right)-\gamma\left(\frac{s}{2A}+\varsigma_0\frac{D_0}{A},k\right)}{\gamma\left(\frac{s}{2D_0},k\right)-\gamma\left(\frac{s}{2A},k\right)}\left(1+\varsigma_0\frac{2D_0}{s}\right)^{-k}=F\,. \qquad (5)$$

Here $\gamma(x,n)$ is the incomplete gamma function

$$\int_0^a\xi^{\mu}e^{-\xi}d\xi=\gamma(a,\mu+1)\,, \qquad (6)$$

and $k=\nu-\eta-2$ is the Bayesian estimation parameter.

Two schemes of CFAR detectors are shown in Fig. 1 and Fig. 2.

If one accounts for a priory statistics (1), then threshold generator of Fig. 1 must operate according to nonlinear transfer curve $\varsigma_0(s,F)=Z_0^2(s)\big/2D_0$ obtained from (5). Such curves are shown in Fig. 3 and Fig. 4 (solid curves) for infinite and limited (illustrative) a priory dynamic range of interference variance L=A/D_0 = 10 ($\eta=0$). Dotted lines in Fig. 3 and Fig. 4 show the corresponding threshold level versus *s* when no a priory data is accounted for.

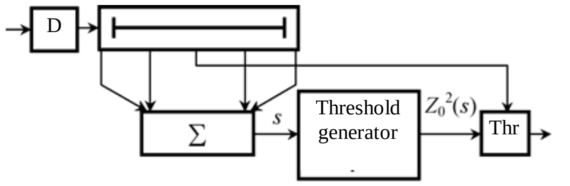


Fig. 1. General block-diagram of CFAR detector

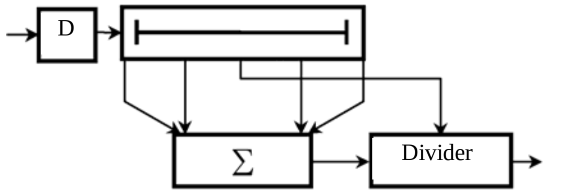


Fig. 2. Simplified block-diagram of CFAR detector

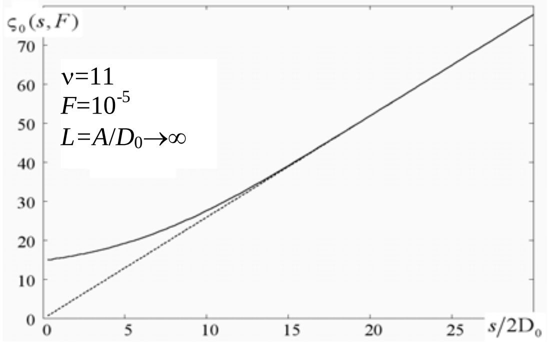


Fig. 3. Transfer curve for the threshold generator for unlimited a priory dynamic range of interference

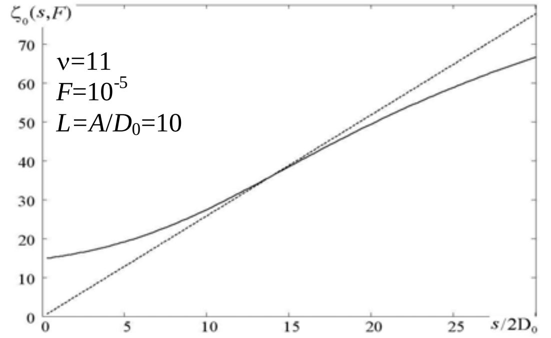


Fig. 4. Transfer curve for the threshold generator for unlimited a priory dynamic range of interference

When the sum s of squared signals outputted from delay line is small, the threshold approaches, in both cases, the level corresponding to the interference absence situation and the adaptation losses disappear. The effect of threshold change is equivalent to automatic gain control when the threshold is kept constant. For the signal operating in conditions of strong interference, the non-linear part of curve of Fig. 3 is insignificant, so, the scheme of Fig. 1 transforms into well known detector of Fig. 2. The curve of Fig. 3 (Fig. 4) can be in advance put into the memory of threshold generator of Fig. 1 and kept unchanged during operation, or be changed in accordance with acquired data for different look directions and ranges.

Adaptive detection probability can be found from

$$D(Z_0,\chi\,|\,s)=\frac{\int_{D_0}^{A}D(Z_0,\chi\,|\,D)p(s\,|\,D)p(D)dD}{\int_{D_0}^{A}p(s\,|\,D)p(D)dD}, \quad (7)$$

where $D(Z_0,\chi\,|\,D)=\exp\left(-Z_0^2/2\chi D\right)=\exp(-\varsigma_0/\chi)$ is the probability of non-adaptive detection and $\chi=1+q^2/2$ is the ratio (detection parameter) of signal+total interference (thermal noise included) variance after optimal processing to that of total interference (TI). Then expression (7) reduces to

$$D(Z_0,\chi|\,s)=\frac{\gamma\left(\frac{s}{2D_0}+\frac{\varsigma_0}{\chi},k\right)-\gamma\left(\frac{s}{2A}+\frac{\varsigma_0}{\chi}\frac{D_0}{A},k\right)}{\gamma\left(\frac{s}{2D_0},k\right)-\gamma\left(\frac{s}{2A},k\right)}\times\left(1+\frac{\varsigma_0}{\chi}\frac{2D_0}{s}\right)^{-k}. \quad (8)$$

Detection probability (8) depends on random value s. General situation can be considered when detection probability (8) is averaged over all possible values of s prior to training. This can be done using unconditional statistics of s

$$D(Z_0,\chi)=\int_0^\infty D(Z_0,\chi\,|\,s)p(s)ds\,, \quad (9)$$

where $p(s)=\int_{D_0}^{A}p(s\,|\,D)p(D)dD$. Probability (9) can be calculated numerically on a computer.

Examples of detection curves for Pareto-optimal CFAR (9) are given in Fig. 5 for F=10^-5^, parameter of Pareto distribution $\eta=0$ (uniform a priory distribution of the interference variance). Detection curves depend on two parameters: Bayesian estimation parameter $k=\nu-\eta-2$ and dynamic range of interference $L=A/D_0$. The L=0 dB corresponds to the case of exactly known variance of interference. The $L=\infty$ dB corresponds to the case when variance of interference varies from 0 dB to ∞ dB.

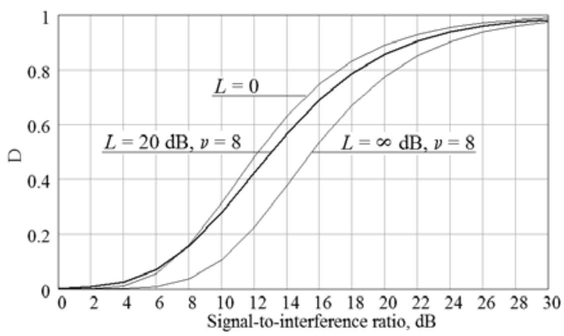


Fig. 5. Detection curves for Bayesian CFAR detector

It can be seen from the curves that detection curves strongly depend on the dynamic range L of external interference and not only on the training sample length ν.

Variants of Bayesian decisions with regard to multi channel reception

In cases of multi channel detection one has to deal with sample matrix of TI, which is the matrix of mutual co-variances of TI in different antenna elements, $\mathbf{S}=\sum_{k=1}^{\nu}\mathbf{Y}_k\mathbf{Y}_k^{*T}$, where $\mathbf{Y}_k$ are the m dimensional snapshot vectors of TI distribution over the antenna array (we assume that signal is absent in this set of data). This matrix can be represented through diagonal one of eigenvalues $\mathbf{\Lambda}_{samp}$ and unitary matrix of eigenvectors $\mathbf{U}_{samp}$ for $\nu\ge m$

$$\mathbf{S}=\mathbf{U}_{samp}\cdot\mathbf{\Lambda}_{samp}\cdot\mathbf{U}_{samp}^{*T}. \quad (10)$$

Vectors $\mathbf{U}_{samp\,i}$ of unitary matrix $\mathbf{U}_{samp}=\left|\mathbf{U}_{samp\,1}\quad\mathbf{U}_{samp\,2}\quad...\quad\mathbf{U}_{samp\,i}\quad...\quad\mathbf{U}_{samp\,m}\right|$ have unit moduli. For an antenna array, their components are steering coefficients. Matrix of sample eigenvalues is diagonal $\mathbf{\Lambda}_{samp} = \mathrm{diag}\{\Lambda_i\}, i = 1,...,m$.

Relation similar to (1) can be applied for a priory probability distribution of TI intensity in this case (after the whitening transform). Distribution of each eigenvalue takes the form

$$p(\Lambda_i) = \Lambda_i{}^{\eta} \Big/ \int_{\Lambda_0}^{B} \Lambda_i{}^{\eta} d\Lambda_i\ ,\ \Lambda_0 \le \Lambda_i < B, \qquad (11)$$

where Λ_0 is the double variance of noise in receiver channels; B is the upper bound of double variance of the combined interference in receiver channels; η is the parameter of variance distribution (if η=0, the distribution is uniform). Since any assumptions cannot be made on the character of dependence between different eigenvalues in advance, they have to be assumed independent

$$p(\mathbf{\Lambda}) = \prod_{i=1}^{m} p(\Lambda_i)\,. \qquad (12)$$

Spreading the scalar case solution onto the multichannel, we can introduce adaptive detection probability similar to (7)

$$D(Z_0, \chi \mid \mathbf{\Lambda}_{samp}) = \frac{\int_{(\mathbf{\Lambda})} D(Z_0, \chi \mid \mathbf{\Lambda}) p(\mathbf{\Lambda}_{samp} \mid \mathbf{\Lambda}) p(\mathbf{\Lambda}) d\mathbf{\Lambda}}{\int_{(\mathbf{\Lambda})} p(\mathbf{\Lambda}_{samp} \mid \mathbf{\Lambda}) p(\mathbf{\Lambda}) d\mathbf{\Lambda}}\,, \qquad (13)$$

and false alarm probability corresponding to signal absence in $D(Z_0, \chi \mid \mathbf{\Lambda})$. The estimate $\hat{\mathbf{\Lambda}}$ of the matrix $\mathbf{\Lambda}$ based on the sample matrix $\mathbf{\Lambda}_{samp}$ is not given in this case. The more informative conditional probability density $p(\mathbf{\Lambda}_{samp} \mid \mathbf{\Lambda})$ replaces the CM estimate in this case. Another variant of utilizing Bayesian statistics reduces to obtainment of the matrix estimate $\hat{\mathbf{\Phi}}(\mathbf{S}) = \mathbf{U}_{samp}(\mathbf{S})\hat{\mathbf{\Lambda}}(\mathbf{S})\mathbf{U}^{*T}_{samp}(\mathbf{S})$ instead of matrix $\mathbf{S}$ given the quadratic loss function. This variant is further referred to as the simplified Bayesian regularization. It is simpler but less effective.

Simplified Bayesian regularization of CM estimate

In this paper, we limit consideration by the simplified Bayesian statistics. Applying Bayesian statistics to the eigenvalue estimates, we can express them as follows

$$\hat{\mathbf{\Lambda}}(\mathbf{S}) = \frac{\int_{[\Lambda_0 B]} \mathbf{\Lambda}\, p[\mathbf{\Lambda}_{samp} \mid \mathbf{\Lambda}] p(\mathbf{\Lambda}) d\mathbf{\Lambda}}{\int_{[\Lambda_0 B]} p[\mathbf{\Lambda}_{samp} \mid \mathbf{\Lambda}] p(\mathbf{\Lambda}) d\mathbf{\Lambda}} = \mathrm{diag}\{\hat{\Lambda}_i\}. \qquad (14)$$

Independent sample eigenvalues have chi-square distributions

$$p[\mathbf{\Lambda}_{samp} \mid \mathbf{\Lambda}] = \prod_{i=1}^{m} p(\Lambda_{samp\,i} \mid \Lambda_i) = \prod_{i=1}^{m} \frac{1}{\Lambda_i{}^{\nu}\Gamma(\nu)} \Lambda_{samp\,i}^{\nu-1} e^{-\Lambda_{samp\,i}/\Lambda_i} \qquad (15)$$

and

$$d\mathbf{\Lambda} = \prod_{i=1}^{m} d\Lambda_i\ . \qquad (16)$$

Then, integrals in numerator and denominator in (14) become products of integrals. After reducing them as before we obtain

$$\hat{\Lambda}_i = \Lambda_{sampi} \frac{\gamma(\Lambda_{sampi}/\Lambda_0, k) - \gamma(\Lambda_{sampi}/B, k)}{\gamma(\Lambda_{sampi}/\Lambda_0, k+1) - \gamma(\Lambda_{sampi}/B, k+1)}, \qquad (17)$$

where $\gamma(x,n)$ is the incomplete gamma function (6), $k = \nu - \eta - 2$ is the Bayesian estimation parameter, ν is equal to the number of snapshot vectors used for estimating the CM (training), and $\Lambda_{samp\,i}$ is the sample value of *i*th eigenvalue. Then, Bayesian estimate of CM is expressed as

$$\hat{\mathbf{\Phi}}(\mathbf{S}) = \mathbf{U}_{samp}(\mathbf{S})\hat{\mathbf{\Lambda}}(\mathbf{S})\mathbf{U}^{*T}_{samp}(\mathbf{S}). \qquad (18)$$

This matrix always has the inverse matrix and no additional artificial regularization technique has to be applied to ensure that inverse CM exists (even for $\nu < m$). The latter is guaranteed by the dependence (17). If all the sample eigenvalues tend to very small values compared to own receiver's noise, their Bayesian estimates tend to the receiver's noise level. This dependence is the same as that shown in Fig. 3, except that value $\Lambda_{samp\,i}/\Lambda_0$ of relative sample eigenvalues substitute for relative sample variance $s/2D_0$. Bayesian covariance matrix estimate proposed here has the property of fast convergence.

Convergence of different adaptive algorithms by signal-to-noise ratio

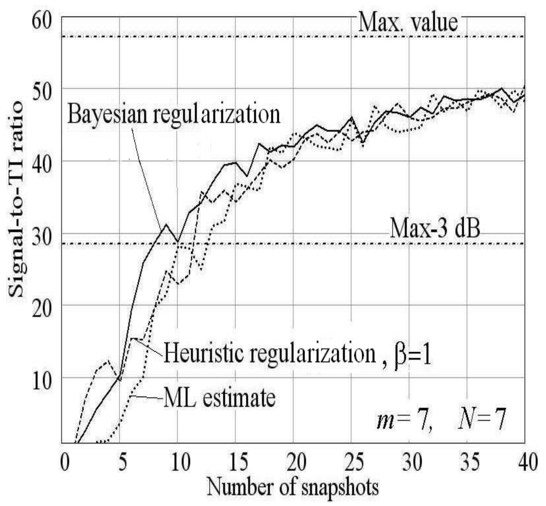
Using computer simulation, different CM estimates were compared by the convergence rate provided that anticipated and actual signal directions are matched.

ML estimate of CM corresponds to $\mathbf{S}/\nu$ (ν is the number of training snapshot vectors). Bayesian regularization of the CM estimate was done using (16). Heuristic regularization was done by adding the matrix $\mathbf{I} \cdot \beta/\nu$ to the ML estimate.

Two linear antenna arrays were simulated with the number of elements *m*=10 and *m*=7, the signal direction corresponded to the aperture normal. Signal power was 10 dB relative to receiver's noise (noise level Λ_0 assumed to be unit). Number of sources of active interference *N*=7, their angular positions were -47°, -41°, -34°, -26°, 21°, 31°, and 47° off the normal direction. Power of all interference sources was equal to 10 dB relative to receiver's noise. The case of absent external interferences *N*=0 was considered too. Output signal-to-TI ratio was calculated as $q^2 = \left|\mathbf{X}^{*T}\hat{\mathbf{\Phi}}^{-1}\mathbf{X}\right|^2 \Big/ (\mathbf{X}^{*T}\hat{\mathbf{\Phi}}^{-1}\mathbf{\Phi}\hat{\mathbf{\Phi}}^{-1}\mathbf{X})$, where $\mathbf{\Phi}$ is the known CM, which is not exactly equal to its estimate $\hat{\mathbf{\Phi}}$.

The transient responses of adaptive spatial filter averaged over 21 realizations versus number of training snapshot vectors for m=7 and m=10 for the cases of present (N=7) and absent (N=0) external interferences are presented in Fig. 6 – Fig. 8. Horizontal lines mark the maximum achievable signal-to-TI ratio (given the exactly known covariance matrix) and its half (3 dB loss) level.

According to simulation results, the regularization (both simplified Bayesian and heuristic) doesn't accelerate convergence compared to ML estimate (Fig. 6 a) if the number of antenna array elements m is equal or less than number of interference sources N, but the simplified Bayesian regularization eliminates the losses on adaptation if external interferences are absent (Fig. 6, b).



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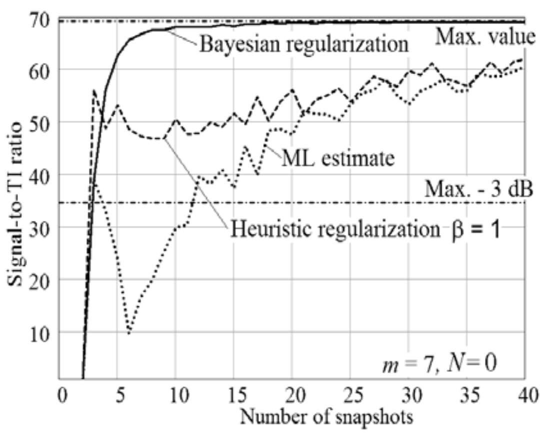

b

Fig. 6. Transient responses of adaptive matched filter

If the number m of antenna array elements is larger than number of interference sources N, Bayesian (Pareto-optimal) regularization allows adaptation algorithm to converge to a solution faster (Fig. 7 a) than in case of ML estimate and its heuristic regularization for $\beta = 1$. Again, in case of absent external interferences adaptation losses are eliminated by the simplified Bayesian regularization (Fig. 7, b).

In order to put heuristic regularization closer to the simplified Bayesian one, we investigated parameter β and have found its optimal value: $\beta = \nu \Lambda_0$. We refer to this case as the best heuristic regularization. To set this parameter in the best way, information on the receiver's (system's) noise level is necessary as in case of simplified Bayesian regularization. The best heuristic regularization gave the result very close to that of simplified Bayesian one (Fig. 8 a). Nevertheless, in case when interferences are absent the adaptation losses were eliminated maximally only using simplified Bayesian regularization (Fig. 8 b).

In case of stronger than 10 dB interference, the maximum signal-to-TI ratio level during our simulation lowered, and the transient process of signal-to-TI ratio stabilization shortened for both Bayesian (Pareto-optimal) and the best heuristic regularization.

Conclusion

The results of paper show one of possible ways of improving the quality of adaptive detection by means of more full utilization of a priory information on interferences for both simplified Bayesian and heuristic regularization [14]. This allows easing requirements to the necessary volumes of training snapshot vectors compared to those presented in [11 – 13] (on the basis of ML estimate of CM). The widely used ML estimates of CM without regularization demonstrate pure results in a number of cases (of weak interferences). Introduction of Bayesian a priory statistics in form of generalized Pareto distributions is effective for both multi-dimensional problems (STAP) and one-dimensional problem (CFAR). The best heuristic regularization is based on the same a priory data about receiver's noise as the Bayesian one. The use of the best heuristic regularization provides good results too, though it's simpler than Bayesian one. Using regularization, either Bayesian or the best heuristic, one may not switch off adaptation when external interferences are absent that is necessary [18] in case of ML statistics.

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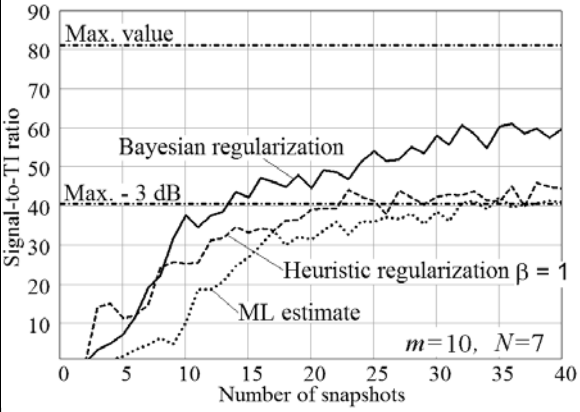

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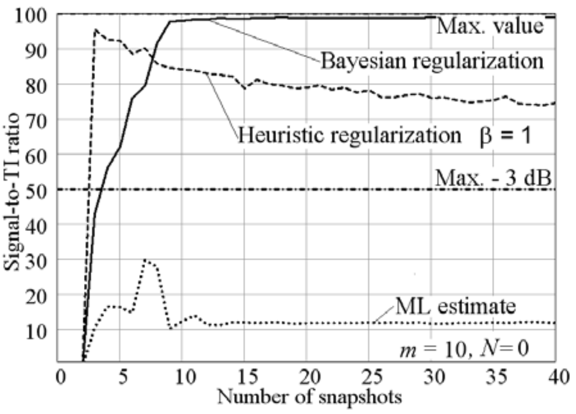
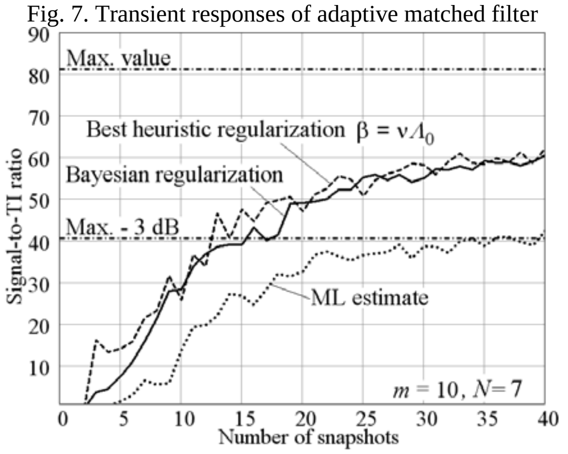


b

Fig. 7. Transient responses of adaptive matched filter



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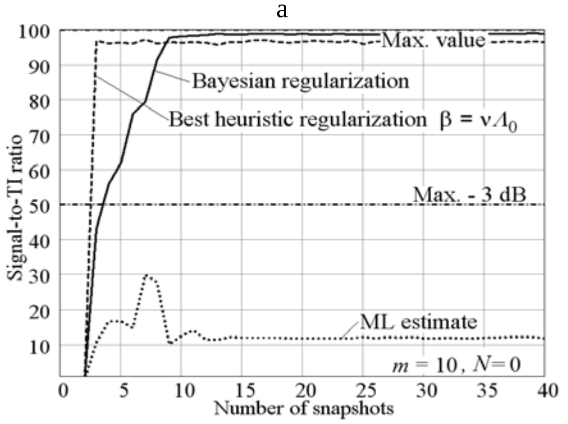

b

Fig. 8. Transient responses of adaptive matched filter

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