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COMPARISON OF THE METHODS OF PROCESSING THE RESULTS OF DEPTH DOSE DISTRIBUTION MEASUREMENTS INITIATED BY ELECTRON BEAM

The work is dedicated to comparison methods of processing the results of the depth dose distributions measurements to determine the practical range of electrons. The sets of test data were obtained by modeling the depth dose distributions with use Monte Carlo method. The accuracy of the calculation method is determined by the mean square error of processing results the sets of test data. The comparison of computational methods of processing the measurement results was performed in the paper. The result of measurements differs in the sizes of the array of data being processed and types of the functions which are used for approximation the data. Comparison the accuracy methods may leads to selection of computational method for determining the practical range of electrons for computational dosimetry of electron radiation.

Keywords*: computational dosimetry, depth dose distribution, electron radiation, simulation by Monte Carlo method, practical range.*

1. Introduction

Planning and control of the irradiation process is carried on the basis of dosimetry measurements. In common practice of the radiation-technological centers, measurements of depth dose distributions are performed with application dosimetric wedge or stack according to the standard dosimetry method of electron radiation [1 – 3].

This type of measurements determine the spatial characteristics of depth dose as well as electron beam practical range R_p.

The standards [2, 3] describe the formal procedures for determining the spatial characteristics of the dose distribution.

Definition the R_p value is at [1]. The practical range R_p is defined as point where the tangent at the steepest point (the inflection point) on the almost straight descending portion of the depth versus absorbed dose curve meets the extrapolated bremsstrahlung background. In accordance with this definition the value R_p can be calculated from relations:

$$x_p = \arg\max_{x\in[Rm,Ro]} (-D'(x)) \text{ or } D''(x_p) = 0; \quad (1)$$

$$R_p = R(x_p);$$
$$R(x) = x - \frac{D(x) - D_{rad}}{D'(x)}, \quad (2)$$

where x_p – the steepest point (the inflection point) on descending part of the dose distribution curve.

R_m – the depth at which dose distribution curve has a maximum value,

R_0 – continuous slowing-down approximation range of electrons,

$D(x)$ – the depth-dose distribution of electron radiation,

$D'(x)$, $D''(x)$ – the first and second derivatives of the depth-dose distribution,

D_{rad} – value of the extrapolated bremsstrahlung background.

The value x_p can be determined with use the second derivative $D''(x)$ as solution of equation (1), either by numerical methods of determining the position of an unconditional minimum of the first derivative $D'(x)$ on the almost straight descending part of the dose distribution. Computing method of determining the value R_p represented by the relations (1) and (2) requires knowledge of first and second derivatives of the depth-dose distribution curve.

However, the measurements results of depth-dose distribution are usually provide as the set of discrete data. Consequently, the formal procedures for determining the practical range R_p are connected with solving of incorrect mathematical tasks. The various types of approximations of tabular data are used for obtaining quasi solutions of incorrect mathematical task. A common method of approximation, which is used in the practice of radiation sterilization centers, is the method of using a linear approximation of data in the field of deep recession

depending on the dose. The methods utilizing an approximation of measurement results in the area of decline (recession) of the depth-dose distributions with polynomials of various degrees (DELEN) were described and tested [4, 5].

In particular, the polynomial of 4-th degree was applied in calculation procedure [5]. However, the traditional method and the proposed methods of polynomial approximation can be used only for a small part of the measurements results, where the proposed approximation is possible. For example, for linear approximation the depth range on the decline of the depth-dose distribution in which the relative value of the dose varies from 0.8 to 0.2 maximum dose can be used.

Evidently, more complete recovery (receiving) information which contains the measurements results of the depth-dose distribution is possible only by using computational methods based on physical models of electron transfer of radiation in matter. Interesting approach was proposed by Lazurik and Pochynok [6] to use of approximation the measuring results of the depth-dose distributions with software tool EMID [7]. EMID software implements a semi-empirical model of the dose distribution of a monoenergetic electrons beam which normally incident on a semi-infinite target.

The method of Parameter Fitting of Semi-Empirical Model (PFSEM) is used in this approach, for processing the measurement results obtained with dosimetric wedge [6, 8]. Herewith, the parameters of semi-empirical model are the following: electron energy (E_0) and displacement of initial point (dX) on depth-dose curve. These parameters ensure of minimum square deviation Q^2 between normalized calculated data and measurement data:

– for fitting the one parameter

$$Q^2(E_0) = \sum_{i=1}^{N} [D_e(x_i, E_0) - D_i]^2 ,$$

– for fitting the two parameters

$$Q^2(dX, E_0) = \sum_{i=1}^{N} [D_e(x_i + dX, E_0) - D_i]^2 . \quad (3)$$

Here $D_e(x, E)$ describes a dose by electrons with energy E on the distance x from surface target, N – the numbers of spatial points in one set of measurements, (D_i , x_i) - normalized measurement results for a set of dose values and spatial coordinates at the measuring points i = 1 .... N.

The method of coordinate descent was used for determination with prescribed accuracy the value of energy E_0 and displacement the initial point dX for the depth-dose curve. Approbation of computational method PFSEM was successfully performed in processing the results of measurements with using of dosimetric wedges in the Institute of Nuclear Chemistry and Technology, Warsaw, Poland [9 – 11].

Formulation of the problem

The determination of the practical range Rp are connected with solving of incorrect mathematical task, therefore, the accuracy of the calculation results depends on methods and type of functions which are used to approximate the measurement results of the depth-dose distribution of electron radiation. In this work a comparison of different methods of processing of measurement results, which have different sizes of data area and type of functions used for approximation of discrete data. The sets of test examples of the depth-dose distribution for the statistical estimates of the random errors of the methods are calculated. The accuracy of the methods is defined as the statistical estimate of the uncertainty in results of data processing.

Obtaining sets of test data

The data sets of the depth-dose distributions calculated by method Monte-Carlo in the detailed physical model of the passage of electron radiation in matter were used for comparison. The computing block "Monte Carlo" in software ModeRTL [12] was applied. Depth-dose distributions into aluminum target for electrons with energy 10 MeV, were calculated.

The values of dose $D_e(x)$ were obtained into 50 space points, which uniformly covered the interval of depths x, from the surface of the target up to R_0 continuous slowing-down approximation range of electrons with energy 10MeV. As a result, 26 statistically independent data sets of the depth-dose distributions were obtained. 10^3 electron trajectories are used for the test cases simulation. Fig.1 shows two histograms of the depth-dose distributions from test cases. The depth-dose curve is shown in figure for comparison.

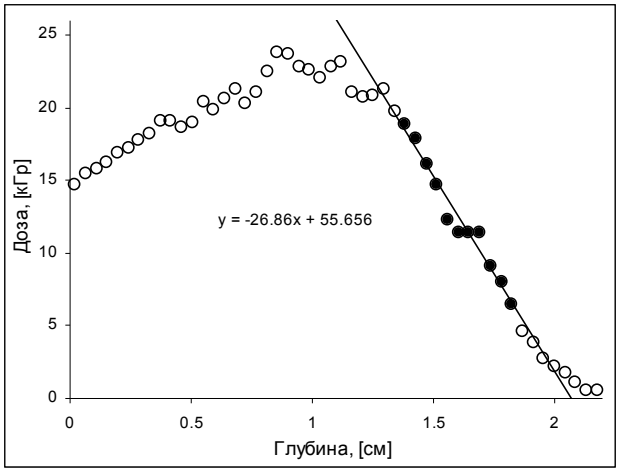

Fig. 1. The examples of statistically independent test cases for the depth-dose distributions

Statistical analysis of the sets test cases

The study of values of the random error of depth dose distributions in test cases was performed. Fig. 2 shows the statistical characteristics of random errors in the dose depending on the depth. Points on the fig. 2 describe the relation of the values of the mean-square deviation to the average dose over the entire depth of field, obtained by statistical processing of 26 test cases.

Curve 1 presented on the fig. 2 represents approximation of a polynomial of the 4-th degree of discrete data in the form of points. Curve 2 describes the ratio of the magnitude value of the standard deviation to the value of dose as function of the depth in target. Curve 2 is calculated on the basis of values approximating polynomial (curve 1) and the values of the depth dose distribution (dotted curve 3).

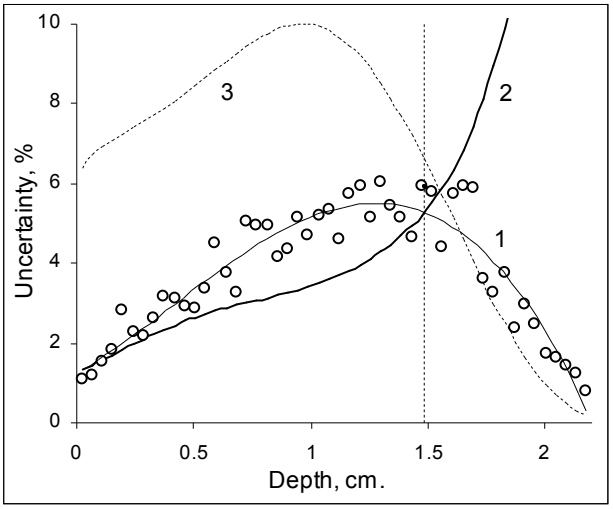

Fig. 2. The uncertainty (standard deviation) the dose values for the sets of the test cases

The fig. 2 shows that the value of uncertainty dose changes significantly with depth and there is a strong distinction to the dependencies represented by curves 1 and 2, from the linear dependence. It means that for a set of test cases, the dose of random errors cannot be described by using simple models like "constant absolute error" or "constant relative error".

The statistical evaluation of the probability density of random error of the dose in the set of test cases was performed. The values of dose deviations for each depth was normalized to the value of standard deviation of the dose at this depth (see Fig. 2, curve1).

The histogram in Fig. 3 shows the probability density function of the relative dose deviations, which was obtained by statistical processing of a set of test cases. For comparison, the fig. 3 shows the normal distribution with parameters (0.1) as the solid curve. As can be seen from the fig. 3, the distribution of dose deviations for the test cases can be assumed as normal probability distribution.

Comparison of computational methods for determining the electrons practical range on the base of processing the test data sets

The following methods were selected for performing comparison:

– linear approximation of depth dose distribution in a limited diapason of the dose values,

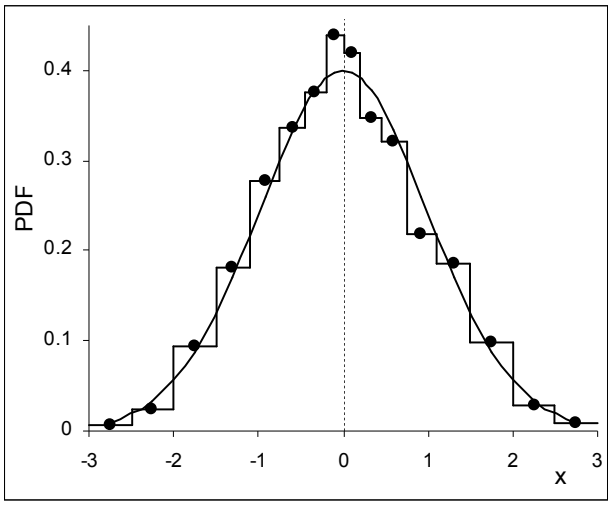

Fig. 3. The probability density function of the relative deviation of the dose

– approximation of the depth-dose distribution in a limited diapason of the dose values using the polynomials of 4th degree,

– parametric fitting of semi-empirical model to the depth-dose distribution in the depths area, where the values of doses were determined.

By comparing the methods, it was selected the sets of dose values from different areas with symmetrical boundaries relatively values the dose $D(x_p)$ at the inflection point x_p of the depth dose distribution. On the basis of numerical studies of depth dose distributions $D(x)$ in the aluminum target irradiated with electrons energy of 10 MeV, it can be assumed

$$K_p = \frac{D(x_p)}{D_{max}} \approx 0.50,$$

where

$$D_{max} = \max_{x \in [Rm,Ro]} (D(x)) \qquad (4)$$

The following values are presented in table 1: K_{min} values for lower and K_{max} for upper boundaries of the doses values.

The column N_p shows the number dose values, which belong to the data treatment area (number of processed nodes). Region A_1 has the maximum number of nodes (Np = 27), because it contains the entire

set of data on the decline of deep dose distribution. Region A_4 has a minimum number of nodes (Np = 6) to approximate polynomial of the 4th degree, because the number of nodes to be processed should be more than five.

Table 1

Area	K_{min}	K_{max}	N_p	R_P-Line	R_P-Pol
A_1	0	1	27	--	2.024
A_2	0.2	0.8	12	2.036	2.017
A_3	0.25	0.75	10	2.030	2.015
A_4	0.3	0.7	6	2.024	2.014
A_5	0.4	0.6	4	2.018	--

Table 1 shows practical range R_P of electrons (in units of [cm]) the calculated on based processing of the depth-dose distributions, which has been obtained with low statistical error (<0.05%), with use the Monte Carlo method.

The processing of the dose distributions was performed using a linear approximation (data in column R_P-Line) and the approximation by polynomials of 4th degree (R_P-Pol column). For example, one of methods for determining the values of a practical range of electrons on the base of processing the depth-dose distribution of electrons, is illustrated in Fig 1. This method uses a linear approximation of data in the A_2 area. The linear equation which describes the tangent to the depth-dose distribution at the inflection point, is shown in the Fig.1. This allows to calculate the value of the practical range of electrons (R_P = 2.072 cm.) for the images shown of the test case.

Average values (column Av) and standard deviation (column σ) values of the practical range, calculated using a set of test cases, are presented in the table 2. The value of uncertainty for processing results (column Unc) was calculated as the ratio of standard deviation to the average value of practical range.

Table 2

	Polinom			Line		
Area	Av. [cm.]	σ [cm.]	Unc., %	Av. [cm.]	σ [cm.]	Unc. %
A_1	2.02	0.028	1.40	--	--	--
A_2	1.96	0.064	3.27	2.048	0.042	2.02
A_3	1.94	0.059	3.04	2.038	0.050	2.43
A_4	1.91	0.088	4.59	2.022	0.076	3.74
A_5	--	--	--	2.058	0.293	14.3

As can be seen from table 2, the standard deviation of values for practical range of electrons is increased with the decreasing of data processing region. Comparing the data in tables 1 and 2 shows that the average values of practical ranges of electrons, produced using linear approximation (column Line) and approximation of polynomials of 4^{th} degree (column Polinom), in satisfactorily agreement within the statistical uncertainty of the calculation results. The results of processing a set of test cases, by parametric fitting method the semi-empirical model to the depth-dose distributions, are presented in the table 3.

Table 3

	E_1	E_0	dX	R_P
Av-stc	10.11	9.91	-0.032	2.021
σ	0.089	0.248	0.038	0.021
Unc., %	0.877	2.50	1.54	1.05
Av-ddd	10.11	9.93	-0.03	2.023
MC	10.10	9.98	-0.02	2.024

The values of fitting parameters were determined for each test case:

E_1 – energy of the electrons, with one-parameter fitting of semi-empirical model, E_0,

dX – electrons energy and displacement of initial point on depth-dose curve for two-parameter fitting of semi-empirical model.

The value of practical range R_P for each test case, was calculated by the empirical formula (A_3), which is given in Appendix. Mean values, standard deviations and statistical uncertainty of fitting parameters and values of practical range, represented in rows with Av-stc, σ and Unc labels, respectively, were calculated using a full set of test cases.

For comparison, in the row labeled with Av-ddd, are indicated the values of the fitting parameters and the value of the practical range of electrons for depth dose distribution, obtained by averaging the dose distributions for a set of test cases. Furthermore, in MC line, it was shown the characteristics of depth-dose distribution, which was obtained by Monte Carlo method with using the detailed physical model and with small statistical error (<0.05%). The comparison of uncertainty value for practical range, which is calculated by different computational methods should considered a significant change in the random error of dose values with depth (see. fig. 2).

Table 4 shows the average values of dose (column Δ_{Av}) and random error dose (column Δ_{Av}) for different areas of A_i, which are used for data approximation.

Coefficients η_P of propagation of random errors of processed data to the calculation results, obtained by

different methods are presented in columns η_P {Line} and η_P {Pol}.

$$\eta_p(A_i) = \frac{\sigma(A_i)}{R_p(A_i)} \Big/ \frac{\Delta_{Av}(A_i)}{D_{Av}(A_i)}. \quad (5)$$

Table 4

Area	D_{Av}, [kGy]	Δ_{Av}, [kGy]	η_P {Line}	η_P {Pol}
A_1	12.27	0.65	--	0.26
A_2	11.34	0.73	0.32	0.51
A_3	11.34	0.76	0.37	0.45
A_4	11.34	0.78	0.55	0.67
A_5	11.34	0.84	1.92	--
A_{tot}	15.44	0.58	--	--

The data for calculating the coefficients $\eta_p(A_i)$ were taken from table 2 (column Av and σ). Data of column R_P in table 3 were used for the calculation of the coefficient $\eta_p(A_{tot})$ - with method of parametric fitting by semi-empirical model to the depth-dose distributions,

$$\eta_p(A_{tot}) \approx 0,28.$$

As follows from the data presented in tables 2 and 3, the average values of a practical range, obtained by different computing methods, are in satisfactory agreement within the standard deviations. As follows from the data presented in tables 2 and 3, the average values of a practical range, obtained by different computing methods, are in satisfactory agreement within the standard deviation.

Comparison coefficients of propagation of random errors of processed data $\eta_p(A_i)$ (see Table 4) with the coefficient $\eta_p(A_{tot})$ obtained by the two-parameter fitting of semi-empirical model, allows grounded to recommend PFSEM-method for computer dosimetry of electron radiation in radiation technologies.

Conclusions

A comparison of various computational methods for determining the practical range of the electrons on the basis of processing of the results of measurements of the deph-dose distribution of electrons radiation, were performed.

The set of test cases for depth-dose distributions were calculated by the Monte-Carlo method in the detailed physical model for this purpose. The accuracy of the computational method was defined as the mean square error for the set of test cases. Compared computational accuracy of measurement results of processing methods, which vary in size and field processed data types of functions used to approximate the data. The results of the comparison makes possible formulation of scientifically substantiate recommendation of two-parametric method of fitting a semi-empirical model (PFSEM method) for computational dosimetry of electron irradiation in radiation processing technologies.

Appendix

Empirical relationship for connection of practical range RP with the parameters of the semi-empirical model of the absorption energy of electrons

A geometric interpretation of the displacement parameter dX in the method of two-parameter fitting, allow us to write the relation [10, 11]:

$$R_p = R_p(E_0) - dX \quad (A1)$$

where $R_p(E)$ - value R_p as function of electron energy E obtained on the base of the semiempirical model of electrons energy deposition,

E_0, dX – the values of model parameters defined by PFSEM method.

The series of calculations on the base of the semi-empirical model of electrons energy deposition was performed for obtaining empirical formulas describing the dependence of $R_p(E)$.

Values of the electrons energy E were selected in the field of relativistic energies from 1 MeV to the border of the estimated accuracy of the semi-empirical model 20MeV.

Depth-dose distribution are obtained from semi-empirical model, derivatives are calculated by the methods of numerical differentiation with minimal values R_p did not exceed 0.1%.

The values of $R_p(E)$ approximated using quadratic functions:

$$R_p(E) = 6*10^{-5}*E^2 + 0.2064*E - 0.0641. \quad (A2)$$

As follows from formulas (6), the contribution of the quadratic term in this energy range is not great. Therefore, it is of interest linear approximation.

$$R_p(E) = 0.2076*E - 0.0683. \quad (A3)$$

In empirical formulas (A2) and (A3) the amount of energy E is in units of MeV, the value of R_p is obtained in cm. The errors of empirical formulas do not exceed 0.01 cm in the energy region from 2 to 20 MeV.

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ПОРІВНЯННЯ МЕТОДІВ ОБРОБКИ РЕЗУЛЬТАТІВ ВИМІРЮВАНЬ ГЛИБИННИХ РОЗПОДІЛІВ ДОЗИ ЕЛЕКТРОННОГО ВИПРОМІНЮВАННЯ

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Робота присвячена порівнянню методів обробки результатів вимірювань глибинних розподілів дози електронного випромінювання для визначення практичного пробігу електронів. Тестові набори даних отримують моделюванням глибинних розподілів дози методом Монте-Карло. Точність обчислювального методу визначають по середньоквадратичній помилки результатів обробки тестових наборів даних. Порівнюють обчислювальні методи обробки результатів вимірювань, які відрізняються розмірами області оброблюваних даних і видами функцій, які використовуються для апроксимації цих даних. На підставі порівняння точності методів обґрунтовуються рекомендації по вибору обчислювальних методів визначення практичного пробігу електронів для комп'ютерної дозиметрії електронного випромінювання.

Ключові слова: *комп'ютерна дозиметрія, глибинний розподіл дози, електронне випромінювання, моделювання методом Монте Карло, практичний пробіг.*

СРАВНЕНИЕ МЕТОДОВ ОБРАБОТКИ РЕЗУЛЬТАТОВ ИЗМЕРЕНИЙ ГЛУБИННЫХ РАСПРЕДЕЛЕНИЙ ДОЗЫ ЭЛЕКТРОННОГО ИЗЛУЧЕНИЯ

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Работа посвящена сравнению методов обработки результатов измерений глубинных распределений дозы электронного излучения для определения практического пробега электронов. Тестовые наборы данных получают моделированием глубинных распределений дозы методом Монте-Карло. Точность вычислительного метода определяют по среднеквадратичной ошибке результатов обработки тестовых наборов данных. Сравнивают вычислительные методы обработки результатов измерений, которые отличаются размерами области обрабатываемых данных и видами функций, используемых для аппроксимации этих данных. На основании сравнения точности методов обосновываются рекомендации по выбору вычислительных методов определения практического пробега электронов для компьютерной дозиметрии электронного излучения.

Ключевые слова*: компьютерная дозиметрия, глубинное распределение дозы, электронное излучение, моделирование методом Монте Карло, практический пробег.*