

UDC 621.391, 681.375

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FIRING ARTILLERY BATTERY AND IMPACT POINT GRID VALUE ANALYTICAL CALCULATIONS BASED ON SERIES OF MEASUREMENTS BY COUNTER ARTILLERY RADAR

Estimation method of impact points' area and firing battery location by results of counter artillery radar measurements is developed based on trajectory prognosis in real time the shell's flight which lies in algorithm synthesis that provide required accuracy of prognosis.

Keywords: artillery shell, counter-battery fight, radar station.

Introduction

Statement of an issue. Counter artillery radars that provide spotting shells in a flight, firing pieces position finding, impact points' area positioning and own artillery fire adjustment are used for detection of batteries' firing positions. These radars perform spotting shells on the initial stage of a flight and series of measurements of projectile current position. Detection of weapon pieces position area and possible impact area is based on approximation of measured results of shell's coordinates and prolongation of their trajectory. Prognosis of shell's flight trajectory can be made:

- based on the fast-time scale computational solution of the differential equation system that describes movement of its centre of mass;
- based on analytic expressions received under different hypothesis of the gravity force and aerodynamic force pattern of change.

As the main hardship of trajectory prolongation lies in ambiguity of shell's ballistic coefficient, the trajectory prognosis in real time the shell's flight lies in algorithm synthesis that provide required accuracy of impact points and firing battery location prognosis.

Review of last research and publications. An issue of the trajectory prognosis in real time the shell's flight was raised in [1–5]. Prognosis algorithm is based on approximation of trajectories with cubical parabola where constant coefficients were calculated due to known shell's design factors, air parameters and results of navigation task solution by on-board computing aids. Estimation method of impact points' area and firing battery location lies in approximating parabola values calculation manner based on external trajectory measurements that was not examined before is proposed in the article.

Research mission statement is counter artillery radar is a system for measuring parameters of shells, mines, rockets and other ballistic objects (hereinafter – shells) trajectory in spherical measurement coordinate system OOX₀Y₀Z₀ that are used to compute their coordinates and traverse speed.

The aim of the article is to develop estimation method of impact points' area and firing battery location based on counter artillery radar measurements.

Basic section

The task of research is to estimate firing battery coordinates and impact point by results of counter artillery radar measurements session of tilt angle ε , azimuth γ and distance to the object \mathcal{D} made during designated period of time $\Delta t_n \in [t_{n0}; t_{n3}]$.

The following succession of given task solution is proposed:

- shell's position computation in measurement coordinate system.
- estimation of true values of computed shell's position.
- approximation of shell's trajectory leg with cubical parabola and estimation of its coefficients' values.
- estimation of firing battery location.
- estimation of shell's initial velocity, gun tube canting angle and product of ballistic coefficient and drag force coefficient.
- estimation of impact point coordinates.

1. Shell's position computation in measurement coordinate system.

Shell's coordinates (x_{0i}, y_{0i}, z_{0i}) in measurement coordinate system $O_o X_o Y_o Z_o$ during designated period of measurement time $\Delta t_n \in [t_{n0}, t_{n3}]$ at discrete moments $t_{ni} = t_{n0} + i \Delta T_n$, $i = \overline{0, I}$ are estimated using formulas:

$$x_{0i} = \mathcal{D}_i \cos \varepsilon_i \cos \gamma_i;$$

$$y_{0i} = \mathcal{D}_i \sin \varepsilon_i;$$

$$z_{0i} = \mathcal{D}_i \cos \varepsilon_i \sin \gamma_i,$$

where \mathcal{D}_i – measured distance from the radar to the shell;

ε_i – measured shell's tilt angle;

- γ_i – measured shell’s azimuth;
 - t_{n_0} – moment of measurements start;
 - ΔT_n – counter artillery radar discrete measurement pitch;
 - Δt_n – duration of measurements session;
 - I – number of measurements in the session;
 - n – measurements session reference;
 - I – measurement reference in the session.
2. Estimation of true values of computed shell’s position.

Values of shell’s centre of mass position vector components including stochastic components resulting from random measurement system errors are obtained as the result of the shell’s flight observation during designated period of measurement time.

Estimation of position vector components true values is made by least-squares method to get steady interdiction point of forecasted shell’s trajectory with horizontal plane OXZ. Equalizing of computed shell’s centre of mass coordinates values during the period of measurement time in horizontal plane OOX0Z0 is done based on linear approximation of points (x_{0i}, z_{0i}) :

$$z_{0i}^* = z_{00}^* + K_z x_{0i}^* .$$

This line will determine vertical plane OXY (see fig. 1) where shell’s trajectory is located and its interdiction angle value ψ with vertical plane OOX0Y0 is estimated using the formula:

$$\Psi = \arctg \frac{z_{0K}^* - z_{00}^*}{x_{0K}^* - x_{00}^*} ,$$

where $(x_{00}^*, z_{00}^*), (x_{0K}^*, z_{0K}^*)$ – coordinates of points corresponding to beginning and end of equalized section.

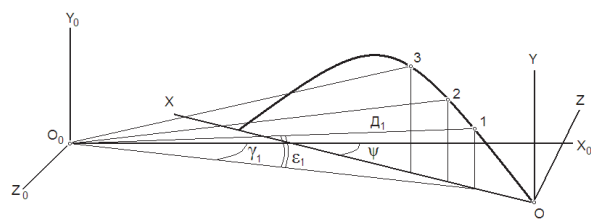


Fig. 1. Constraint of shell’s trajectory parameters and measurement coordinate system

Rectilinear start coordinate system is introduced to describe shell’s trajectory OXYZ: beginning of coordinate system – point O is located on the end of gun tube, axis OX – horizontal and is directed to the gun aim point; axis OY – is directed straight up; axis OZ – complements coordinate system to the right group of three.

Then transition from coordinates in measurement coordinate system OOX0Y0Z0 to start coordinate system OXYZ is made using formulas:

$$x_i^* = \frac{x_{0i}^*}{\cos \Psi}, \quad y_i = y_{0i}, \quad z_i = 0, \quad \cos \Psi \neq 0 .$$

At this stage of computation the location of start coordinate system OXYZ beginning relative to measurement coordinate system OOX0Y0Z0 is unknown.

Equalizing of computed shell coordinates values at measurement section in vertical plane OXY is done based on cubical approximation of points (x_i^*, y_i) :

$$y_i^* = b_0 + b_1 x_i^* + b_2 (x_i^*)^2 + b_3 (x_i^*)^3 .$$

Values of constant coefficients $b_i \quad i = 0, 1, 2, 3$ of this cubical parabola are gained by least-squares method.

3. Approximation of shell’s trajectory leg with cubical parabola and estimation of its coefficients’ values.

Shell’s centre of mass trajectory in vertical plane OXY in start coordinate system is possible to approximate with cubical parabola with satisfactory accuracy:

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 .$$

As a result of equalizing of shell’s centre of mass coordinates values in start coordinate system OXY at measurement section of trajectory we will gain coordinates of five points (x_i^*, y_i^*, z_i^*) , $i = 1, 2, 3, 4, 5$ and we will equate 5 combined linear equations where the location of start coordinate system OXY beginning relative to measurement coordinate system x_0 and constant coefficients a_0, a_1, a_2, a_3 (in the record the superscript * is missed) are unknown:

$$\begin{cases} y_1 = a_0 + a_1 (x_1 - x_0) + a_2 (x_1 - x_0)^2 + a_3 (x_1 - x_0)^3 ; \\ y_2 = a_0 + a_1 (x_2 - x_0) + a_2 (x_2 - x_0)^2 + a_3 (x_2 - x_0)^3 ; \\ y_3 = a_0 + a_1 (x_3 - x_0) + a_2 (x_3 - x_0)^2 + a_3 (x_3 - x_0)^3 ; \\ y_4 = a_0 + a_1 (x_4 - x_0) + a_2 (x_4 - x_0)^2 + a_3 (x_4 - x_0)^3 ; \\ y_5 = a_0 + a_1 (x_5 - x_0) + a_2 (x_5 - x_0)^2 + a_3 (x_5 - x_0)^3 . \end{cases}$$

We will decrease scale of the system and after transformations we will result in:

$$\begin{cases} \Delta y_{21} = a_2 (x_2^2 - x_1^2) + a_3 (x_2^3 - x_1^3) - \\ - 3a_3 x_0 (x_2^2 - x_1^2) + (a_1 - 2a_2 x_0 + 3a_3 x_0^2) \Delta x_{21} ; \\ \Delta y_{31} = a_2 (x_3^2 - x_1^2) + a_3 (x_3^3 - x_1^3) - \\ - 3a_3 x_0 (x_3^2 - x_1^2) + (a_1 - 2a_2 x_0 + 3a_3 x_0^2) \Delta x_{31} ; \\ \Delta y_{41} = a_2 (x_4^2 - x_1^2) + a_3 (x_4^3 - x_1^3) - \\ - 3a_3 x_0 (x_4^2 - x_1^2) + (a_1 - 2a_2 x_0 + 3a_3 x_0^2) \Delta x_{41} ; \\ \Delta y_{51} = a_2 (x_5^2 - x_1^2) + a_3 (x_5^3 - x_1^3) - \\ - 3a_3 x_0 (x_5^2 - x_1^2) + (a_1 - 2a_2 x_0 + 3a_3 x_0^2) \Delta x_{51} ; \end{cases}$$

$$\begin{aligned}\Delta x_{21} &= x_2 - x_1; & \Delta x_{31} &= x_3 - x_1; \\ \Delta x_{41} &= x_4 - x_1; & \Delta x_{51} &= x_5 - x_1; \\ \Delta y_{21} &= y_2 - y_1; & \Delta y_{31} &= y_3 - y_1; \\ \Delta y_{41} &= y_4 - y_1; & \Delta y_{51} &= y_5 - y_1.\end{aligned}$$

Let's present these 4 combined linear equations matrix-like:

$$Y = M_X A; \quad Y = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{pmatrix}; \quad A = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix};$$

$$M_X = \begin{pmatrix} X_{11} & X_{12} & X_{13} & X_{14} \\ X_{21} & X_{22} & X_{23} & X_{24} \\ X_{31} & X_{32} & X_{33} & X_{34} \\ X_{41} & X_{42} & X_{43} & X_{44} \end{pmatrix};$$

$$A_1 = a_2; \quad A_2 = a_3; \quad A_3 = -3a_3 x_0;$$

$$A_4 = a_1 - 2a_2 x_0 + 3a_3 x_0^2;$$

$$Y_1 = \Delta y_{21}; \quad Y_2 = \Delta y_{31}; \quad Y_3 = \Delta y_{41}; \quad Y_4 = \Delta y_{51};$$

$$X_{11} = x_2^2 - x_1^2; \quad X_{12} = x_2^3 - x_1^3;$$

$$X_{13} = x_2^2 - x_1^2; \quad X_{14} = \Delta x_{21};$$

$$X_{21} = x_3^2 - x_1^2; \quad X_{22} = x_3^3 - x_1^3;$$

$$X_{23} = x_3^2 - x_1^2; \quad X_{24} = \Delta x_{31};$$

$$X_{31} = x_4^2 - x_1^2; \quad X_{32} = x_4^3 - x_1^3;$$

$$X_{33} = x_4^2 - x_1^2; \quad X_{34} = \Delta x_{41};$$

$$X_{41} = x_5^2 - x_1^2; \quad X_{42} = x_5^3 - x_1^3;$$

$$X_{43} = x_5^2 - x_1^2; \quad X_{44} = \Delta x_{51};$$

where M_X – 4 by 4 square matrix with known values of X_{ij} elements;

Y – 4 by 1 vector with known values of Y_i elements;

A – 4 by 1 vector and its A_i elements is necessary to evaluate. Also its necessary to evaluate x_0 .

If position vectors $\bar{r}(t_1); \bar{r}(t_2); \bar{r}(t_3); \bar{r}(t_4); \bar{r}(t_5)$ that characterize location of shell's centre of mass in measurement coordinate system are not collinear, then determinant $D = \det M_X \neq 0$ corresponds to M_X matrix.

Values of A_i elements of A vector are equated using the formula:

$$A_i = \frac{D_i}{D}; \quad i = 1, 2, 3, 4.$$

where determinant D_i is gained from $D = \det M_X$ with replacement of i column with Y column.

After equation of A_i elements of A vector the values of x_0, a_1, a_2, a_3 are equated with formulas:

$$a_2 = A_1; \quad a_3 = A_2; \quad x_0 = -\frac{A_3}{3A_2};$$

$$a_1 = A_4 - \frac{A_3}{3A_2}(2A_1 + A_3).$$

Let's put obtained expressions into cubical parabola formula to get expression for a_0 coefficient:

$$\begin{aligned}a_0 &= y_1 - \left(A_4 - \frac{A_3}{3A_2}(2A_1 + A_3) \right) \left(x_1 + \frac{A_3}{3A_2} \right) - \\ &- A_1 \left(x_1 + \frac{A_3}{3A_2} \right)^2 - A_2 \left(x_1 + \frac{A_3}{3A_2} \right)^3.\end{aligned}$$

4. Estimation of firing battery location.

Estimation of firing battery coordinates in measurement coordinate system is made using formulas:

$$x_{cb} = x_0 \cos \Psi; \quad z_{cb} = x_0 \sin \Psi; \quad y_{cb} = a_0.$$

5. Estimation of shell's initial velocity, gun tube canting angle and product of ballistic coefficient and drag force coefficient.

Constant coefficients a_i $i=0, 1, 2, 3$ of cubical parabola can be expressed through initial conditions of shell's flight y_0, θ_0, V_0 , its design factors S, m , average value of air density ρ_c and acceleration of gravity g_c using formulas:

$$a_0 = y_0; \quad a_1 = \text{tg } \theta_0;$$

$$a_2 = -\frac{g_c}{2 V_{x0}^2};$$

$$a_3 = -\frac{g_c \rho_c}{3 V_{x0}^2} C_x \sigma;$$

$$\sigma = \frac{S}{2m}; \quad V_{x0} = V_0 \cos \theta_0;$$

$$g_c = \frac{g_0 R^2}{(y_0 + R)(y_m + R)}; \quad g_0 = 9,81 \text{ м/с}^2;$$

$$R = 6371110 \text{ м};$$

$$\rho_c = \frac{\rho_0}{\beta} \frac{1}{y_m - y_0} [\exp(-\beta y_m) - \exp(-\beta y_0)];$$

$$\rho_0 = 1,225875 \text{ кг/м}^3; \quad \beta = 0,000141 \text{ м}^{-1},$$

where y_m – max height of shell's trajectory.

Estimation of initial conditions of shell's flight values is made:

– initial velocity vector inclination angle using the formula:

$$\theta_0 = \arctg \left(A_4 - \frac{A_3}{3A_2}(2A_1 + A_3) \right);$$

– initial velocity vector length using the formula:

$$V_0 = \sqrt{-\frac{g_c}{2 A_1 \cos^2 \theta_0}}.$$

Let's divide a_3 coefficient by a_2 coefficient and after transformation we will obtain formula for equation of $C_x \sigma$ coefficients product using the formula:

$$C_x \sigma = \frac{3 A_2}{2 A_1 \rho_c}$$

6. Estimation of impact point coordinates.

To find coordinates of impact point it's necessary to find its flight range in start coordinate system OXY. For that reason it's necessary to solve cubic equation (*). To find roots of cubic equation (*) we will get so called reduced equation:

$$X^3 + pX + q = 0;$$

$$p = -3 \frac{1}{\rho_c \sigma C_x} \left(\frac{v_0^2}{g_c} \sin \theta_0 \cos \theta_0 + \frac{1}{4 \rho_c \sigma C_x} \right);$$

$$q = -\frac{1}{\rho_c \sigma C_x} \left(\frac{p}{3} + 3 \frac{v_0^2}{g_c} y_0 \right).$$

Discriminant of reduced cubic equation has the appearance:

$$D_K = -\frac{1}{(\rho_c \sigma C_x)^2} \left[\frac{1}{\rho_c \sigma C_x} \left(\frac{v_0^2}{g_c} \sin \theta_0 \cos \theta_0 + \frac{1}{4 \rho_c \sigma C_x} \right)^3 - \frac{1}{4} \left(\frac{p}{3} + 3 \frac{v_0^2}{g_c} y_0 \right)^2 \right].$$

As $D_K < 0$ then cubic equation has three real solutions that can be found using Cardano formula:

$$X_2 = -\frac{u_1 + u_2}{2} + \frac{u_1 - u_2}{2} i \sqrt{3};$$

$$X_1 = u_1 + u_2;$$

$$X_3 = -\frac{u_1 + u_2}{2} - \frac{u_1 - u_2}{2} i \sqrt{3};$$

$$u_1 = \sqrt[3]{-\frac{q}{2} + \sqrt{D_K}}; \quad u_2 = \sqrt[3]{-\frac{q}{2} - \sqrt{D_K}};$$

$$D_K = \left(\frac{p}{3} \right)^3 + \left(\frac{q}{2} \right)^2.$$

Via replacement $x_k = X_k - \frac{a_2}{3a_3}$; $k = 1, 2, 3$ from

X_k we will obtain solution x_k of this cubic equation.

In particular case when $a_0 = 0$ the cubic equation (*) can be presented as:

$$x (a_1 + a_2 x + a_3 x^2) = 0.$$

Then, three real roots will be solution of the equation:

$$x_1 = 0,$$

$$x_{2,3} = \frac{-a_2 \pm \sqrt{a_2^2 - 4a_3 a_1}}{2a_3}, \text{ or}$$

$$x_{2,3} = -\frac{3}{4} \frac{1 \pm \sqrt{1 + 5 \frac{1}{3} \frac{\rho_c}{g_c} C_x \sigma V_0^2 \sin \theta_0 \cos \theta_0}}{\rho_c C_x \sigma}.$$

Zero value corresponds to shell's origin of trajectory or the beginning of coordinate system OXY.

Positive real root will correspond to shell's flight range in coordinate system OXY:

$$x_2 = \frac{3}{4} \frac{\sqrt{1 + 5 \frac{1}{3} \frac{\rho_c}{g_c} C_x \sigma V_0^2 \sin \theta_0 \cos \theta_0} - 1}{\rho_c C_x \sigma}.$$

Coordinates of impact point in measurement coordinate system are computed using formulas:

$$x_{\text{ТП}} = (x_0 \pm x_2) \cos \Psi;$$

$$z_{\text{ТП}} = (x_0 \pm x_2) \sin \Psi;$$

$$y_{\text{ТП}} = 0.$$

In the mentioned formulas "+" sign should be selected if shell is flying from the counter artillery radar and "-" sign should be selected if shell is flying to the counter artillery radar.

It's necessary to mention if shell's flight is observed on the initial trajectory leg the estimation of firing battery location is more precise and estimation of impact point coordinates is less precise. For further adjustment of impact point coordinates it's necessary to make measurements on subsequent trajectory legs. The most precision can be obtained based on the measurements of trajectory parameters on the shell's descent or final trajectory leg. Recent essential while using counter artillery radar for own artillery fire adjustment.

Conclusions

Developed mathematical apparatus tool is based on analytic relation for equation of firing battery location and impact point coordinates. It can be used for control of equations made based on tabular integration of differential equation system that describes shell's centre of mass movement/ it also can be used for estimation of product of ballistic coefficient and drag force coefficient indeterminate value which is necessary for numerous calculations.

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Надійшла до редколегії 14.11.2016

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**АНАЛІТИЧНИЙ МЕТОД РОЗРАХУНКУ ЗНАЧЕНЬ
КООРДИНАТ СТРІЛЯЮЧОЇ БАТАРЕЇ
І ТОЧКИ ПАДІННЯ СНАРЯДА ПО СЕРІЇ ВИМІРЮВАНЬ
РАДІОЛОКАЦІЙНОЮ СТАНЦІЄЮ КОНТРБАТАРЕЙНОЇ БОРотьБИ**

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Розроблено метод оцінки області точок падіння снарядів та місця розташування стріляючої батареї за результатами вимірювань в радіолокаційних станціях контрбатареїної боротьби (РЛС КББ) на основі вирішення задачі прогнозування траєкторії в масштабі реального часу польоту снарядів, яка полягає в синтезі алгоритмів, що забезпечують необхідну точність прогнозування.

Ключові слова: артилерійський снаряд, контрбатареїна боротьба, радіолокаційна станція.

**АНАЛИТИЧЕСКИЙ МЕТОД РАСЧЕТА ЗНАЧЕНИЙ
КООРДИНАТ СТРЕЛЯЮЩЕЙ БАТАРЕИ
И ТОЧКИ ПАДЕНИЯ СНАРЯДА ПО СЕРИИ ИЗМЕРЕНИЙ
РАДИОЛОКАЦИОННОЙ СТАНЦИЕЙ КОНТРБАТАРЕЙНОЙ БОРЬБЫ**

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Разработан метод оценки области точек падения снарядов и местоположения стреляющей батареи по результатам измерений в радиолокационных станциях контрбатареиной борьбы (РЛС КББ) на основе решения задачи прогнозирования траектории в масштабе реального времени полета снарядов, которая заключается в синтезе алгоритмов, обеспечивающих необходимую точность прогнозирования.

Ключевые слова: артиллерийский снаряд, контрбатареиная борьба, радиолокационная станция.