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SPECTRAL ANALYSIS OF SIGNALS BY ROOT-MIN-NORM METHOD, USING MODIFIED SSA METHOD

The paper considers the problem of efficiency enhancing of spectral analysis of the signals observed against noise via Root-Min-Norm method with data preprocessing by modification of the SSA method. Simulations results are presented that confirm the enhancement of spectral analysis efficiency by Root-Min-Norm method when using the SSA method and its modification.

Keywords: *noise reduction, eigenvector, eigenvalues, eigenbasis, singular vectors, singular values, modified SSA method.*

Introduction

In the modern radioelectronic systems (RES) of information transmission and measuring RES, signals are considered as elements of a certain space, the properties of signals are viewed as space properties and signal transformations are considered as mapping of one space into another space. The mathematical base for such representation is functional analysis. The concepts of spaces, norms, basis allow to formalize the processes related to the transmission and reception of signals [1–2].

The signal processing usually requires the formation of some basis such as the basis of eigenvectors of the covariance matrix (CM) of observations, the power basis, the basis based on the using of Gram-Shmidt orthogonalisation and so on [1].

The eigenvalue decomposition (EVD) of the CM of observations or singular value decomposition of the data matrix is the base of the modern spectral analysis methods. Furthermore, the methods of noise reduction, the methods of signal processing in communication systems with a multi-element antenna array on the transmitting and receiving side (MIMO-multiple input-multiple output system), pattern recognition (recognition of signals with digital modulation or images) methods, methods of parameter estimation of compound signals also use EVD or SVD [1–6].

It should be noted that the Karhunen-Loeve transform (also known as Hotelling transform) is related with EVD and widely used in many fields of data analysis [2]. In the MIMO communication systems the eigenvectors (singular vectors) are used for parallel data transmission, precoding, beamforming (for example, in the radio SC3800 of Silvus StreamCaster™), the channel state estimation.

The subspace – based methods are related with separation of eigenvectors of covariance matrix on corresponding to signal subspace and orthogonal subspace (usually such subspace is called noise subspace). However, the term orthogonal subspace is more appropriate because such subspace is orthogonal complement of signal subspace.

In the case of antenna array signal processing (spatial spectral analysis) the MUSIC and Min-Norm methods use the orthogonality between the steering vectors of signals and noise subspace eigenvectors. The ESPRIT uses the property of invariance relatively to shift of signal subspace [1; 7].

Efficiency of spectral analysis methods becomes worse in the conditions of the low signal-to-noise ratio, small samples. It can be improved using the preliminary signal processing (preprocessing) methods [2; 4–5].

The beamspace transformation, spatial smoothing, forward-backward averaging, noise reduction (signal filtering), Gram-Shmidt orthogonalizaiton, singular spectrum analysis (SSA) [1–6] are the examples of the preliminary processing methods that can be used with spectral analysis methods.

SSA is one of the methods of noise reduction in observation, such as the total least squares (TLS), surrogate data technology, wavelet transformation [4–5].

It is known that efficiency of the Min-Norm (Root-Min-Norm) method depends on estimate of covariance matrix of observation [7].

Therefore, the purpose of the paper is to improve the efficiency of signal spectral analysis by Root-Min-Norm method in the conditions of small samples, using the modified SSA method.

Data model

The sequence of observation results $y(n)$ can be expressed as [8]:

$$y(n) = \sum_{v=1}^{V} x_v(n) + e(n) = s(n) + e(n)\,,\ n = 1,\ldots,N\,, \quad (1)$$

where $x_v(n) = \alpha_v \sin(\omega_v n + \varphi_v)$ is the v-th harmonic component, $e(n)$ is the white Gaussian noise, α_v is the amplitude, $\omega_v = 2\pi f_v$ is the frequency, and φ_v is the phase of v-th component. It is assumed, that φ_v are the random independent values equally spaced on the interval $[0, 2\pi)$, $\omega_v \in [0, \pi)$, and $e(n)$ is the white Gaussian noise with zero mean and variance σ^2. The estimates of frequencies of V signal harmonic components $\omega_v, v = 1, \cdots, V$ must be obtained based on the observation $\{y(n)\}_{n=1}^{N}$.

The data matrix with the Hankel structure formed from the input sample $y(n)$ of size N can be presented as [2; 4; 6]

$$\mathbf{Y} = \begin{bmatrix} y(1) & y(2) & \cdots & y(N-m+1) \\ y(2) & y(3) & \cdots & y(N-m+2) \\ \vdots & \vdots & \cdots & \vdots \\ y(m) & y(m+1) & \cdots & y(N) \end{bmatrix}. \quad (2)$$

It can be seen that columns of the matrix are the vectors (segments) of size $m > 2V$ $\mathbf{y}(n) = [y(n) \ldots y(n+m-1)]^T$, $()^T$ denotes transposing, $n = 1, \ldots, K$, $K = N - m + 1$. In the nonlinear dynamics the similar segmentation of time series is called embedding in the phase space [4].

The sample CM of observation can be obtained as $\hat{\mathbf{R}} = \frac{1}{K}\sum_{n=1}^{K} \mathbf{y}(n)\mathbf{y}^T(n) = \frac{1}{K}\mathbf{Y}\mathbf{Y}^T$. Let us introduce the basis from eigenvectors $\{\mathbf{u}_i\}_{i=1}^{m}$. The CM $\mathbf{R}$ can be written in the form $\mathbf{R} = \sum_{i=1}^{m} \gamma_i \mathbf{u}_i \mathbf{u}_i^T$. The singular value decomposition of the data matrix $\mathbf{Y}$ yields by analogy [2; 4–6]:

$$\mathbf{Y} = \sum_{q=1}^{m_y} \mu_q \mathbf{u}_q \mathbf{v}_q^T, \quad (3)$$

where $m_y \le \min\{m, K\}$ is the rank of matrix $\mathbf{Y}$, μ_q are the singular values, $\hat{\mathbf{u}}_q$ are the left singular vectors, and $\hat{\mathbf{v}}_q$ are the right singular vectors of data matrix $\mathbf{Y}$. The left and right singular vectors form the orthonormal basis of row space and column space of matrix $\mathbf{Y}$ respectively.

The eigendecomposition of the sample CM $\hat{\mathbf{R}}$ yields [4–5]:

$$\hat{\mathbf{R}} = \sum_{q=1}^{m} \hat{\gamma}_q \hat{\mathbf{u}}_q \hat{\mathbf{u}}_q^T = [\hat{\mathbf{U}}_s \hat{\mathbf{U}}_n] \begin{bmatrix} \hat{\mathbf{\Lambda}}_s & \mathbf{0} \\ \mathbf{0} & \hat{\Lambda}_n \end{bmatrix} \begin{bmatrix} \hat{\mathbf{U}}_s^T \\ \hat{\mathbf{U}}_n^T \end{bmatrix}, \quad (4)$$

where $\hat{\gamma}_1 > \hat{\gamma}_2 > \ldots > \hat{\gamma}_{\hat{V}}$ are the signal-subspace eigenvalues, $\hat{\gamma}_{\hat{V}+1} \approx \hat{\sigma}^2, \ldots, \hat{\gamma}_K \approx \hat{\sigma}^2$ and $\hat{\gamma}_{K+1} \approx \hat{\gamma}_{K+2} \approx \ldots \hat{\gamma}_m \approx 0$ are the noise-subspace eigenvalues, $\hat{\mathbf{U}}_s = [\hat{\mathbf{u}}_1 \cdots \hat{\mathbf{u}}_{\hat{V}}]$ is the $m \times \hat{V}$ matrix constructed with signal-subspace eigenvectors, $\hat{\mathbf{U}}_n$ is the $m \times (m - \hat{V})$ matrix constructed of noise-subspace eigenvectors, $\hat{\mathbf{\Lambda}}_s$ is the diagonal matrix that contains $\hat{V}$ signal eigenvalues, $\hat{\mathbf{\Lambda}}_n$ is the diagonal matrix of $m - \hat{V}$ noise-subspace eigenvalues, and $\hat{V}$ is the estimate of the number of harmonic components.

The realization of eigenstructure methods requires spectral decomposition (EVD) of the CM $\hat{\mathbf{R}}$ or SVD of data matrix $\mathbf{Y}$. The Root-Min-Norm estimates the frequencies of harmonic components as roots of polinomial [1]:

$$P_{rmn}(z) = \mathbf{a}^H(z)\mathbf{\Pi}^{\perp}\mathbf{e}_1\mathbf{e}_1^T\mathbf{\Pi}^{\perp}\mathbf{a}(z), \quad (5)$$

where $\mathbf{e}_i$ is the $M \times 1$ vector with all zeros elements except the first one, equal to unity, $\mathbf{\Pi}^{\perp} = \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^T$ is the projector on the orthogonal subspace (noise subspace), $\mathbf{a}(z) = [1, z, \ldots, z^{M-1}]^T$, $z = \exp(j\omega)$.

Signal spectral analysis by Root-Min-Norm, using modified SSA method

The fundamentals of SSA method are connected with signal processing [6] and nonlinear dynamic [2; 4]. The modified SSA method is presented in [5]. The realization of the modified SSA method requires the following steps: 1) arrange the sample $\{y(n)\}_{n=1}^{N}$ into the Hankel data matrix $\mathbf{Y}$; 2) compute the EVD of $\hat{\mathbf{R}}$ or SVD of data matrix $\mathbf{Y}$; 3) select $\hat{V}$ signal-subspace singular values and corresponding singular vectors; 4) construct the data matrix filtered from observation noise $\mathbf{Y}_{filt.} = \sum_{q=1}^{\hat{V}} (\hat{\mu}_q - \hat{\sigma})\hat{\mathbf{u}}_q \mathbf{v}_q^T$, where $\hat{\sigma}$ is the estimate of noise variance; 5) construct the filtered sample of time series $y_{filt.}(n)$ based on using the hankelisation operator (i.e. by averaging the elements of the matrix $\mathbf{Y}_{filt.}$ located on cross diagonals of the matrix).

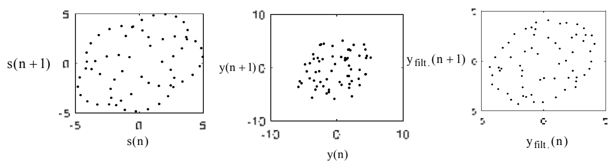
It should be noted that as mentioned in [6] the noise reduction procedure can be repeated several times to improve the results.

It can be seen from equation for $\mathbf{Y}_{filt.}$ that estimate $\hat{\sigma}$ influences on the efficiency of filtration of input sequence from observation noise.

The usual estimate of noise variance is defined as $\hat{\sigma}^2 = (1/(m-\hat{V}))\,\mathrm{trace}(\hat{\Lambda}_n)$ [8]. The feature of the estimate is the components of noise added to signal eigenvalues are not taken into account. Such situation determines the fact that this estimate is an underestimate of noise variance. In the paper the estimate of noise variance $\hat{\sigma}_{mod}{}^2 = \hat{\sigma}_1^2 / (1 - \hat{V}/K)$ is used [5], where $\hat{\sigma}_1^2 = \hat{\sigma}^2 + (1/K)\sum_{q=1}^{\hat{V}} (\gamma_q \hat{\sigma}^2)/(\gamma_q - \hat{\sigma}^2)$. Such estimate is based on the using of G-analysis (also known as the random matrix theory) [9; 10].

The joint realization of the Root-Min-Norm method and the SSA method requires the performing of the following sequence of the steps: 1) construct $\mathbf{Y}_{ssa.}$ based on $\mathbf{y}_{filt.}(n)$; 2) calculate EVD of the matrix $\mathbf{R} = \mathbf{Y}_{ssa}\mathbf{Y}^{T}_{ssa.}$; 3) realize the Root-Min-Norm method, using noise-subspace eigenvectors of $\mathbf{R}$.

In order to consider the results of modified SSA method application together with Root-Min-Norm, let us assume that the signal consists of two equal power harmonic components with frequencies $f_1 = 0.2$ Hz and $f_2 = 0.212$ Hz. The so-called phase portraits are presented on fig. 1. The signal-to-noise ratio (SNR) was 4 dB, $N = 64$. Fig. 1, a corresponds to the signal, the fig. 1, b corresponds to the input sequence and fig. 1, c corresponds to the sequence filtered by modified SSA method.



a b c

Fig. 1. Phase portraits

It can be seen that structure of the point set of the filtered sequence phase portrait of saves the property of signal better than one of the input sequence.

Simulation was performed using the signal described above (i.e. the frequency separation is less than Rayleigh resolution limit). The number of signal components $\hat{V} = 4$ because of using the real signal model [8]. The signal-to-noise ratio (SNR) was defined as $10\log_{10}(\sum_{v=1}^{V} \alpha_v^2 / \sigma^2)$. A total of $L = 1000$ independent simulation runs were performed to obtain each simulated point. Root-mean square error (RMSE) of frequency estimation of the signal harmonic components was averaged on the number of signal components [5].

We compare the performances of Root-Min-Norm for initial data (without using SSA), for data after using SSA method and after using modified SSA method. Fig. 2 shows the experimental RMSE's of frequency estimation of the compared methods versus the SNR. The segment size was $m = 18$.

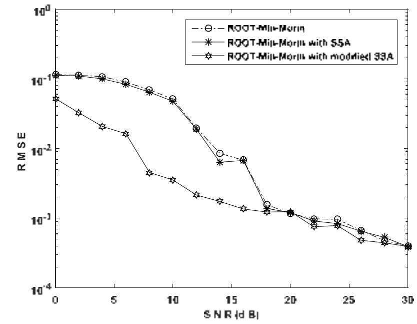


Fig. 2. RMSE of frequency estimation of signal harmonic components by Root-Min-Norm versus SNR, $m = 18$

Fig. 2 shows that using of the modified (improved) SSA method allows to improve the performance of spectral analysis by Root-Min-Norm. In other words, the preprocessing with using the modified SSA method is the most effective for considered case.

Furthermore, the repeated application of the modified SSA method to filtered data allows to additionally improve the performance of spectral analysis. The performance of SSA and modified SSA depends on the of choice of method parameters [2; 5].

In the second case we consider the case of small sample ($m = 53$ and $K = N - m + 1 = 12$).

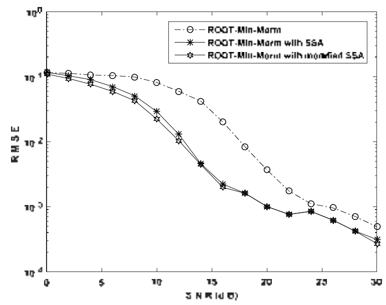


Fig. 3. RMSE of frequency estimation of signal harmonic components by Root-Min-Norm versus SNR, $m = 53$

Fig. 3 shows that in such case the SSA and the modified SSA methods also allow improving the performance of Root-Min-Norm. As one could expect, the threshold SNR in this case is higher than for the conditions of fig. 2.

Conclusion

The paper has considered the joint use of the modified SSA method with Root-Min-Norm method. The modified SSA method uses the noise variance estimate obtained using random matrix theory. The application of the SSA and modified SSA methods allows improving the performance of spectral analysis by Root-Min-Norm method even in the case of a small sample.

It is of interest to generalize the results of paper for the case of antenna array signal processing, for the cases of another bases (such as power basis), for the complex-valued case. Furthermore, it is of interest to use the considered approach for the problem of estimating the parameters of the linear chirp signal, the phase shift keying and the frequency shift keying signals and for related problems.

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СПЕКТРАЛЬНИЙ АНАЛІЗ СИГНАЛІВ МЕТОДОМ ROOT-MIN-NORM З ВИКОРИСТАННЯМ МОДИФІКОВАНОГО МЕТОДУ SSA

В.І. Василишин

У статті розглядається задача підвищення ефективності спектрального аналізу сигналів, що спостерігаються на фоні шуму, методом Root-Min-Norm при попередній обробці даних з використанням модифікованого методу SSA. Представлені результати імітаційного моделювання, що підтверджують підвищення ефективності спектрального аналізу методом Root-Min-Norm при використанні методу SSA та його модифікації.

Ключові слова: *зменшення шуму в спостереженні, власні вектори, власні значення, базис власних векторів, сингулярні значення, сингулярні вектори, модифікований метод SSA.*

СПЕКТРАЛЬНЫЙ АНАЛИЗ СИГНАЛОВ МЕТОДОМ ROOT-MIN-NORM С ИСПОЛЬЗОВАНИЕМ МОДИФИЦИРОВАННОГО МЕТОДА SSA

В.И. Василишин

В статье рассматривается задача повышения эффективности спектрального анализа наблюдаемых на фоне шума сигналов методом Root-Min- Norm при предварительной обработке данных с использованием модифицированного метода SSA. Представлены результаты имитационного моделирования, которые подтверждают повышение эффективности спектрального анализа методом Root-Min- Norm при использовании метода SSA и его модификации.

Ключевые слова: *уменьшение шума в наблюдении, собственные вектора, собственные значения, базис собственных векторов, сингулярные значения, сингулярные вектора, модифицированный метод SSA.*