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THE CALCULATION OF WOUND TUBULAR MANOMETRIC SPRINGS SENSITIVITY OF AVERAGE THICKNESS BY THE SHELLS THEORY METHOD

The new approach to solve the calculation problem of the stress-strain state of the sensing element of the manometric and thermal devices, that is advisable to apply in the design process is considered in this article.

The object of the research is wound manometric spring.

The subject of the research is the stress-strain state of wound manometric springs with different shapes of cross-sections.

The aim of the study is the creation of mathematical model of the object stress-strain state, which allows, in future, to solve the problem of optimization of its design parameters in the automatic calculation algorithm for the automatic design of springs.

As a result of the research, the model of stress-strain state of wound manometric springs was developed, the optimization problem of its parameters taking into account constructive and technological restrictions was formulated and the method to solve this problem was proposed.

The area of use is aircraft instruments designed to measure the pressure of liquids and gases.

Keywords: *mathematical model, stress-strain state, wound tubular manometric spring, optimization of target function.*

Introduction

Wound tubular springs (fig. 1), used to measure high pressures are common sensing elements of manometric devices. Springs designs are the most technological, simple and reliable in operation. The wide application of tubular springs makes important the problem definition of their optimal design.

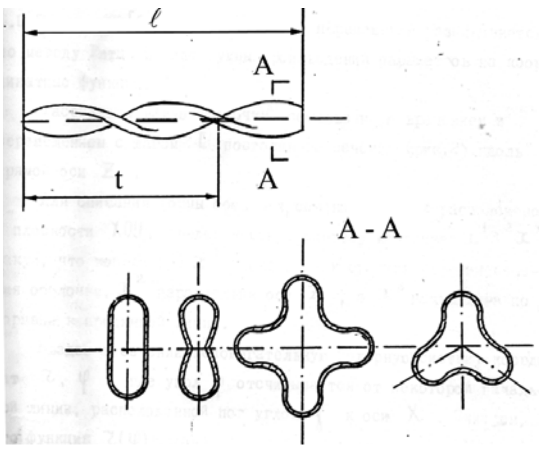

Fig. 1. External view and cross-sections of wound tubular springs, used to measure high pressure

Analysis of the recent researches and publications. The attempts to create a calculation method of stress-strain state of wound manometric spring are made in [1–8; 13; 15].

The difference between this work and works mentioned above is that it offers a calculation method of stress-strain state of wound manometric springs.

The results of the research can be further used to solve the optimization problem of design parameters of springs and to maximize the target function.

The target function is the sensitivity of wound manometric springs at given strength, constructive and technological restrictions.

In this article we propose the adjusted calculation method of wound tubular springs. Here, the spring is regarded as a wound shell. Moreover, in the case when the shell is not thin, the transverse shear strain takes into account.

The aim of the study. The aim of the study is the creating of mathematical model of stress-strain state of wound manometric springs and the statement of the problem to choose in the optimum way its design parameters.

Main material presenting

The methodology presented in this article is based on the shells theory of average thickness, based on S.P. Timoshenko`s model [11], and the required displacement can be found by the Ritz method in the form of the parameters sum of productions on the coordinate functions.

The wound tubular shell of constant cross-section, formed by rotation and displacement with t step along Z-axis is considered in this research (fig. 2).

To describe the shape of the shell, which cross-section is located in the XOY plane**,** introduced such local $\boldsymbol{x^1}$ $\boldsymbol{x^2}$ $\boldsymbol{x^3}$coordinate system, where $\boldsymbol{x^1}$ axis is tangential to the median line of the shell cross-section, x^3 axis is parallel to the Z-axis, and x^2 axis is directed along the normal to the midline.

Moreover, an auxiliary polar r, φ coordinate system is introduced into spring section, where φ angle is measured from an some initial line located at γ angle to X- axis.

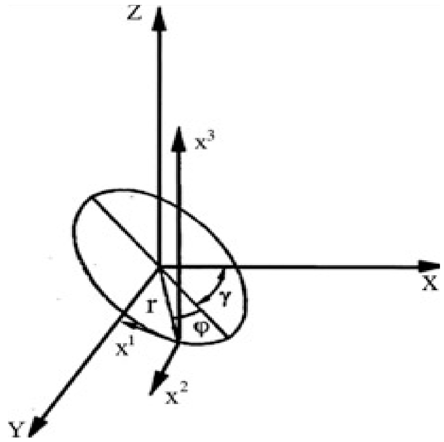

Fig. 2. Axes and angles relative to which the calculation of the stress-strain state of the spring is carrying out

We assume that r (φ) is given.

We assume that x^3 coordinate is equal to the distance of the current cross-section from some initial cross-section. This distance is proportional to the angle of rotation of γ cross-section:

$$x^3 = C \cdot \gamma .$$

Here $C = \frac{t}{2\pi}$, where t is wound step.

Taking into account above mentioned, we obtain the following shape parameterization of the median surface shape of wound shell of arbitrary transverse section.

$$Z = x^3 ;$$
$$X = r(\phi) \cdot \cos(\phi + \frac{x^3}{C}) ; \quad (1)$$
$$Y = r(\phi) \cdot \sin(\phi + \frac{x^3}{C}) .$$

In accordance with the theory of shells of arbitrary shape, the components of the first and second metric tensors of middle surface, and the expressions for Christoffel symbols, constructed according to [10] have the form:

$$a_{11} = 1 ; \quad a_{12} = \frac{r^2}{c} \frac{1}{\sqrt{r^2 + r^{12}}} ; \quad a_{12} = 1 ;$$
$$b_{11} = \frac{r^2 - 2r^{12.} + r^{11} r}{\sqrt{(r^2 + r^{12})^3}} ; \; b_{12} = -\frac{1}{C} ; \quad (2)$$
$$b_{22} = \frac{-r^2}{c^2} \frac{1}{\sqrt{(r^2 + r^{12})}} ;$$
$$\Gamma^1_{11} = -\frac{2r^2}{c^2} \cdot \frac{r' \cdot r(r'' \cdot r - 2r^{12} - r^2}{\sqrt{(r^2 + r^{12})^5}} ;$$
$$\Gamma^2_{11} = -\frac{r}{c^2} \cdot \frac{r' \cdot r(r'' \cdot r - 2r^{12} - r^2)}{\sqrt{(r^2 + r^{12})^2}} ; \quad (3)$$
$$\Gamma^1_{12} = \Gamma^1_{21} = \Gamma^2_{22} = \Gamma^2_{12} = \Gamma^{21} = 0 .$$

In the formulas derivation (2) and (3), it was assumed that the shell is slightly wounded. So we can neglect $\left(\frac{r}{c}\right)$ value in comparison with 1 value.

The r (φ) function, determining the shape of the shell cross-section, can be represented as a segment of Fourier series.

The technique that applied further for improved calculation of wound tubular springs, is based on the modified Timoshenko's model [10], in which the components of u_i displacement vector are represented in the form

$$u_\alpha(x^3) = v_\alpha + x^3(\theta_\alpha + \psi_\alpha) ; \quad (4)$$
$$u_3(x^3) = \omega ,$$

where u_α is the displacement of an arbitrary point of the shell in x^α (α = 1,2) direction

u_3 is the same quantity in x^3 direction

υ^α is the displacement of the middle surface with x^1 x^2 coordinates

$(\phi_\alpha + \theta_\alpha)$ is the normal angle of rotation

at x^1 x^2 point in x^α direction (the normal to the middle surface of the shell under the loads action passes into line. This line is nonoorthogonal to the deformed middle surface.)

ω – is normal displacement at x^1 x^2 point.

The stress-strain state of arbitrary shape shells of average thickness describes the condition of equality to zero of variation of the following functional:

$$I = \frac{1}{2}\int_\Omega (A^{\alpha\beta\gamma}Y_{\alpha\beta}Y_{\gamma\delta} + 2B^{\alpha\beta\gamma}Y_{\alpha\beta}\Phi_{\gamma\delta} + C^{\alpha\beta\gamma\delta}Y_{\alpha\beta}Y_{\gamma\delta})d\Omega +$$
$$+ D^{\alpha\gamma}\psi_\alpha\psi_\gamma - p\omega)d\Omega - \quad (5)$$
$$- \int_g \left[N^\alpha\upsilon_\alpha + M^\alpha(\psi_\alpha - \omega_\alpha - \beta^\nu_\alpha\upsilon_\nu) + Q\omega \right] dg .$$

Here the first integral is taken along the middle surface of the shell, and the second one is taken along its edges.

N^α, M^α and Q values are the specified membrane stresses, moments and shear stresses that distributed along the shell edges.

The tensors of the elastic coefficients are calculated as follows:

$$A^{\alpha\beta\gamma\delta} = 2h\frac{E}{1+\nu}\left[\frac{\nu}{1-\nu}a^{\alpha\beta}b^{\gamma\delta} + \frac{1}{2}(a^{\alpha\gamma}a^{\beta\delta} + a^{\alpha\delta}a^{\beta\gamma})\right] ; \quad (6)$$
$$B^{\alpha\beta\gamma\delta} = \frac{2}{3}h^3\frac{E}{1+\nu}\left[\frac{\nu}{1-\nu}(a^{\alpha\beta}b^{\gamma\delta} + a^{\gamma\delta}b^{\alpha\beta} - 2Ha^{\alpha\beta}a^{\gamma\delta}) +\right.$$
$$+ \frac{1}{2}(2a^{\alpha\gamma}b^{\beta\delta} + 2Ha^{\alpha\gamma}a^{\beta\delta} + a^{\alpha\delta}b^{\beta\gamma} + \quad (7)$$

$$a^{\beta\gamma}b^{\alpha\delta}-2Ha^{\beta\gamma}a^{\alpha\delta})];$$

$$C^{\alpha\beta\gamma\delta}=\frac{h^2}{3}A^{\alpha\beta\gamma\delta};\qquad(8)$$

$$D^{\alpha\gamma}=h\frac{E}{1+\nu}a^{\alpha\gamma}.\qquad(9)$$

Where h is the half thickness of the shell; E is the modulus of elasticity of material; ν is the Poisson's ratio; H is the average curvature of the middle surface; $a^{\alpha\beta}$, $b^{\gamma\delta}$ are matrix coefficients, which are expressed in terms of the coefficients of the first and second quadratic forms, respectively, according to formulas ($\alpha,\beta,\gamma,\delta=1,2$):

$$a^{11}=\frac{a_{22}}{D},\ a^{22}=\frac{a_{11}}{D},\ a^{12}=a^{21}=-\frac{a_{12}}{D};$$

$$D=\det(a_{\alpha\beta})=a_{11}a_{22}-a_{12}^2;$$

$$b^{11}=\frac{b_{22}}{D_b},\quad b^{22}=\frac{b_{11}}{D_b},\quad b^{12}=b^{21}=-\frac{b_{12}}{D},$$

where $$D_b=b_{11}b_{22}-b_{12}^2\qquad(10)$$

As the H is the average curvature of the middle surface, then:

$$2H=b_{11}a^{11}+2b_{11}a^{21}-b_{22}a^{22}.\qquad(11)$$

In the functional (5) $Y_{\alpha\beta}$ and $\Phi_{\alpha\beta}$ values are equal:

$$Y_{\alpha\beta}=\upsilon_{\alpha\|\beta}-b_{\alpha\beta}\omega=\frac{d\upsilon_\alpha}{dx^\beta}-\Gamma^\rho_{\alpha\beta}\upsilon_\rho-b_{\alpha\beta}\omega;\qquad(12)$$

$$\Phi_{\alpha\beta}=\upsilon_{\alpha\|\beta}-(\omega_\alpha+b^\lambda_\alpha\upsilon_\lambda)_{\|\beta}=\frac{d\psi_\alpha}{dx^\beta}-\Gamma^\rho_{\alpha\beta}\psi_\rho+$$

$$+\frac{d\theta_\alpha}{dx^\beta}-\Gamma^\rho_{\alpha\beta}\theta_\rho=\frac{d\psi_\alpha}{dx^\beta}-\Gamma^\rho_{\alpha\beta}\psi_\rho-\frac{\partial^2\omega}{\partial x^\alpha\partial x^\beta}-\frac{db^\lambda_\alpha}{dx^\beta}\upsilon_\lambda-\qquad(13)$$

$$-\frac{d\upsilon_\lambda}{dx^\beta}b^\lambda_\alpha-\Gamma^\rho_{\alpha\beta}\theta_\rho.$$

In formulas given above symbol || is symbol of covariant differentiation in the middle surface metric

$$\theta_1=-\frac{\partial\omega}{\partial x'}-b^{'}_1\upsilon_1-b^2_1\upsilon_2=-\frac{\partial\omega}{\partial x'}-b^\rho_1\upsilon_\rho;$$

$$\theta_2=-\frac{\partial\omega}{\partial x^2}-b^{'}_2\upsilon_1-b^2_2\upsilon_2=-\frac{\partial\omega}{\partial x^2}-b^\rho_2\upsilon_\rho.$$

The average pressure value applied to the spring is equal:

$$P=P^+\mu(+h)+P^-\mu(-h),\qquad(14)$$

where

$$\mu(z)=\det\mu^\nu_\alpha(z);$$

$$\mu^\nu_\alpha=\delta^\nu_\alpha-x^3b^\nu_\alpha.$$

In case the relative thickness of the shell is small and the deformations of transverse shear can be neglected:

$$\mu^\nu_\alpha=\delta^\nu_\alpha\ \text{and}\ B^{\alpha\beta\gamma\delta}=0.$$

Further, we assume that the shell is sufficiently long and that the influence of the ends on the stressed state can also be neglected.

In this case, all the unknown functions depend only on one x^3 coordinate.

We represent the required displacements in the form:

$$\upsilon_1=\sum_{i\in\exists_1}C_i\upsilon^i_1;\quad \omega=\sum_{i\in\exists_3}C_i\omega^i;$$

$$\upsilon_1=\sum_{i\in\exists_1}C_i\upsilon^i_1;\quad \psi_1=\sum_{i\in\exists_4}C_i\psi^i_1;\qquad(15)$$

$$\psi_2=\sum_{i\in\exists_5}C_i\psi^i_2.$$

The ranges of the *i* parameter change are:

$$\exists_k:\ \sum_{S=1}^{k-1}n_S+1\le i\le\sum_{S=1}^{k}n_S,$$

where **(**k = 1,2, ….5).

In (15) C_i coefficients are the required Ritz coefficients. The functions standing for them in the form of multipliers are given coordinate functions that depend on x^3.

Substituting the formulas for displacements in the functional (5) and equating the derivatives of it with respect to Ritz parameters to the zero, we obtain a system of linear algebraic equations:

$$\frac{\partial I}{\partial I}=0=\int_\Omega\left\{\left(A^{\alpha\beta\gamma\delta}\frac{\partial Y_{\alpha\beta}}{\partial C_i}+B^{\alpha\beta\gamma\delta}\frac{\partial\Phi_{\alpha\beta}}{\partial C_i}\right)\cdot Y_{\gamma\delta}+\right.$$

$$+\left(B^{\alpha\beta\gamma\delta}\frac{\partial Y_{\alpha\beta}}{\partial C_i}+C^{\alpha\beta\gamma\delta}\frac{\partial\Phi_{\alpha\beta}}{\partial C_i}\right)\cdot\Phi_{\gamma\delta}+$$

$$\left.+D^{\alpha\gamma}\psi_\gamma\frac{\partial\psi_\alpha}{\partial C_i}-P\frac{\partial\omega}{\partial C_i}\right\}d\Omega\ -\qquad(16)$$

$$-\int_g\left[N^\alpha\frac{\partial\upsilon_\alpha}{\partial C_i}+M^\alpha\frac{\partial}{\partial C_i}(\psi_\alpha-\omega_\alpha-b^\nu_\alpha\upsilon_\alpha)+Q\frac{\partial\omega}{\partial C_i}\right]dg,$$

where $i\in(\exists_1,\exists_2,\exists_3,\exists_4,\exists_5)$ ($\gamma,\delta=1,2$).

Taking into account that i parameter takes values in five regions corresponding to the variation of i_1, i_2, ψ_1, ψ_2 functions we see that the matrix of the system has a block structure, and the right-hand side of the system is a vector consisting of five sub-vectors (fig. 3).

d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	m_1
	d_{22}	d_{23}	d_{24}	d_{25}	m_2
		d_{33}	d_{34}	d_{35}	m_3
			d_{44}	d_{45}	m_4
				d_{55}	m_5

Fig. 3. Redistribution of sub-vectors (coefficients) in stress-strain state matrix

The expressions for the derivatives of functions incoming in to the functional are:

$$\frac{\partial Y_{\alpha\beta}}{\partial C_i}=\begin{cases}\delta_1^{\alpha}\dfrac{\partial \upsilon_{\alpha}^{i}}{\partial x^{\beta}}-\Gamma_{\alpha\beta}^{1}\upsilon_1^i=f_{1\alpha\beta}^{i} & \text{для } i\in\exists_1;\\ \delta_2^{\alpha}\dfrac{\partial \upsilon_{\alpha}^{i}}{\partial x^{\beta}}-\Gamma_{\alpha\beta}^{2}\upsilon_2^i=f_{2\alpha\beta}^{i} & \text{для } i\in\exists_2;\\ -b_{\alpha\beta}\omega^i=f_{3\alpha\beta}^{i} & \text{для } i\in\exists_3;\\ 0 & \text{для } i\in\exists_4;\\ 0 & \text{для } i\in\exists_5;\end{cases} \quad (17)$$

$$\frac{\partial \Phi_{\alpha\beta}}{\partial C_i}=\begin{cases}\dfrac{\partial b_{\alpha}^{1}}{\partial x^{\beta}}\upsilon_1^i-b_{\alpha}^{1}\dfrac{\partial \upsilon_1^i}{\partial x^{\beta}}+\Gamma_{\alpha\beta}^{\rho}b_{\rho}^{1}\upsilon_1^i=\psi_{1\alpha\beta}^{i} & \text{для } i\in\exists_1;\\ \dfrac{\partial b_{\alpha}^{2}}{\partial x^{\beta}}\upsilon_2^i-b_{\alpha}^{2}\dfrac{\partial \upsilon_2^i}{\partial x^{\beta}}+\Gamma_{\alpha\beta}^{\rho}b_{\rho}^{2}\upsilon_2^i=\psi_{2\alpha\beta}^{i} & \text{для } i\in\exists_2;\\ -\dfrac{\partial^2\omega^i}{\partial x^{\alpha}}-\Gamma_{\alpha\beta}^{\rho}\dfrac{\partial \omega^i}{\partial x^{\rho}}=\psi_{3\alpha\beta}^{i} & \text{для } i\in\exists_3;\\ \delta_1^{\alpha}\dfrac{\partial \psi_{\alpha}^{i}}{\partial x^{\beta}}-\Gamma_{\alpha\beta}^{1}\psi_1^i=\psi_{4\alpha\beta}^{i} & \text{для } i\in\exists_4;\\ \delta_2^{\alpha}\dfrac{\partial \psi_{\alpha}^{i}}{\partial x^{\beta}}-\Gamma_{\alpha\beta}^{2}\psi_2^i=\psi_{5\alpha\beta}^{i} & \text{для } i\in\exists_5;\end{cases} \quad (18)$$

$$\frac{\partial \psi_{\alpha}}{\partial C_i}=\begin{cases}0 & \text{для } i\in\exists_1\\ 0 & \text{для } i\in\exists_2\\ 0 & \text{для } i\in\exists_3\\ \psi_1^i & \text{для } i\in\exists_4\\ \psi_2^i & \text{для } i\in\exists_5.\end{cases} \quad (19)$$

As well as for functions:

$$Y_{\gamma\delta}=\sum_{\exists_1}C_K f_{1\gamma\delta}^{K}+\sum_{\exists_2}C_K f_{2\gamma\delta}^{K}+\sum_{\exists_3}C_K f_{3\gamma\delta}^{K}; \quad (20)$$

$$\Phi_{\gamma\delta}=\sum_{\exists_1}C_K\phi_{1\gamma\delta}^{K}+\sum_{\exists_2}C_K\phi_{2\gamma\delta}^{K}+;$$

$$+\sum_{\exists_3}C_K\phi_{3\gamma\delta}^{K}+\sum_{\exists_4}C_K\phi_{4\gamma\delta}^{K}+\sum_{\exists_5}C_K\phi_{5\gamma\delta}^{K}; \quad (21)$$

$$\psi_{\gamma}=\sum_{\exists_4}C_K\psi_{1\gamma\delta}^{K}+\sum_{\exists_5}C_K\psi_{2\gamma\delta}^{K}. \quad (22)$$

Taking into account relations (17–22), we obtain the coefficients of the matrix blocks:

$$d_{11}=\int_{\Omega}\Big\{(A^{\alpha\beta\gamma\delta}f_{1\alpha\beta}^{i}+B^{\alpha\beta\gamma\delta}\phi_{1\alpha\beta}^{i})f_{1\gamma\delta}^{K}+$$
$$+(B^{\alpha\beta\gamma\delta}f_{1\alpha\beta}^{i}+C^{\alpha\beta\gamma\delta}\phi_{1\alpha\beta}^{i})\phi_{1\gamma\delta}^{K}\Big\}d\Omega;$$

$$d_{12}=\int_{\Omega}\Big\{(A^{\alpha\beta\gamma\delta}f_{1\alpha\beta}^{i}+B^{\alpha\beta\gamma\delta}\phi_{1\alpha\beta}^{i})f_{2\gamma\delta}^{K}+$$
$$+(B^{\alpha\beta\gamma\delta}f_{1\alpha\beta}^{i}+C^{\alpha\beta\gamma\delta}\phi_{1\alpha\beta}^{i})\phi_{2\gamma\delta}^{K}\Big\}d\Omega;$$

$$d_{13}=\int_{\Omega}\Big\{(A^{\alpha\beta\gamma\delta}f_{1\alpha\beta}^{i}+B^{\alpha\beta\gamma\delta}\phi_{1\alpha\beta}^{i})f_{3\gamma\delta}^{K}+$$
$$+(B^{\alpha\beta\gamma\delta}f_{1\alpha\beta}^{i}+C^{\alpha\beta\gamma\delta}\phi_{1\alpha\beta}^{i})\phi_{3\gamma\delta}^{K}\Big\}d\Omega;$$

$$d_{14}=\int_{\Omega}\Big\{(B^{\alpha\beta\gamma\delta}f_{1\alpha\beta}^{i}+C^{\alpha\beta\gamma\delta}\phi_{1\alpha\beta}^{i})\phi_{4\gamma\delta}^{K}\Big\}d\Omega;$$

$$d_{15}=\int_{\Omega}\Big\{(B^{\alpha\beta\gamma\delta}f_{1\alpha\beta}^{i}+C^{\alpha\beta\gamma\delta}\phi_{1\gamma\delta}^{K})\phi_{5\gamma\delta}^{K}\Big\}d\Omega;$$

$$d_{22}=\int_{\Omega}\Big\{(A^{\alpha\beta\gamma\delta}f_{2\alpha\beta}^{i}+B^{\alpha\beta\gamma\delta}\phi_{2\alpha\beta}^{i})f_{2\gamma\delta}^{K}+$$
$$+(B^{\alpha\beta\gamma\delta}f_{2\alpha\beta}^{i}+C^{\alpha\beta\gamma\delta}\phi_{2\alpha\beta}^{i})\phi_{2\gamma\delta}^{K}\Big\}d\Omega;$$

$$d_{23}=\int_{\Omega}\Big\{(A^{\alpha\beta\gamma\delta}f_{2\alpha\beta}^{i}+B^{\alpha\beta\gamma\delta}\phi_{2\alpha\beta}^{i})f_{3\gamma\delta}^{K}+$$
$$+(B^{\alpha\beta\gamma\delta}f_{2\alpha\beta}^{i}+C^{\alpha\beta\gamma\delta}\phi_{2\alpha\beta}^{i})\phi_{3\gamma\delta}^{K}\Big\}d\Omega;$$

$$d_{24}=\int_{\Omega}\Big\{(B^{\alpha\beta\gamma\delta}f_{2\alpha\beta}^{i}+C^{\alpha\beta\gamma\delta}\phi_{2\alpha\beta}^{i})\phi_{4\gamma\delta}^{K}\Big\}d\Omega;$$

$$d_{25}=\int_{\Omega}\Big\{(B^{\alpha\beta\gamma\delta}f_{2\alpha\beta}^{i}+C^{\alpha\beta\gamma\delta}\phi_{5\gamma\delta}^{K})\Big\}d\Omega; \quad (23)$$

$$d_{33}=\int_{\Omega}\Big\{(A^{\alpha\beta\gamma\delta}f_{3\alpha\beta}^{i}+B^{\alpha\beta\gamma\delta}\phi_{3\alpha\beta}^{i})f_{3\gamma\delta}^{K}+$$
$$+(B^{\alpha\beta\gamma\delta}f_{3\alpha\beta}^{i}+C^{\alpha\beta\gamma\delta}\varphi_{3\alpha\beta}^{i})\varphi_{3\gamma\delta}^{K}\Big\}d\Omega;$$

$$d_{34}=\int_{\Omega}\Big\{(B^{\alpha\beta\gamma\delta}f_{3\alpha\beta}^{i}+C^{\alpha\beta\gamma\delta}\phi_{3\alpha\beta}^{i})\phi_{4\gamma\delta}^{K}\Big\}d\Omega;$$

$$d_{35}=\int_{\Omega}\Big\{(B^{\alpha\beta\gamma\delta}f_{3\alpha\beta}^{i}+C^{\alpha\beta\gamma\delta}\phi_{3\alpha\beta}^{i})\phi_{5\gamma\delta}^{K}\Big\}d\Omega;$$

$$d_{44}=\int_{\Omega}\Big\{D^{\alpha\gamma}\psi_1^i\psi_1^K+C^{\alpha\beta\gamma\delta}\phi_{4\alpha\beta}^{i}\phi_{5\gamma\delta}^{K}\Big\}d\Omega;$$

$$d_{55}=\int_{\Omega}\Big\{D^{\alpha\gamma}\psi_2^i\psi_2^K+C^{\alpha\beta\gamma\delta}\phi_{5\alpha\beta}^{i}\phi_{5\gamma\delta}^{K}\Big\}d\Omega.$$

In $d_{s\ell}=d_{\ell s}$ blocks (where s,ℓ = 1, 2,…, 5) the i and k indices belong to $\exists_s$ areas, and $\exists_\ell$ respectively ($i\in\exists_s$, $k\in\exists_\ell$).

Let's write the elements of the subvectors of the right-hand side matrix for case of the pressure action and the N axial force applied at the edges of the shell:

$$m_1=\int_g N\upsilon_1^i dg;\quad m_2=0;\quad m_3=\int_{\Omega}P\omega^i d\Omega;$$

$$m_4=m_5=0. \quad (24)$$

In m_s vectors i index belongs to the $\exists_s$ ($i\in\exists_s$ area).

The number of rows in the block-rows consist respectively on 1-5 is n_1, n_2, ..., n_5.

After solving the system of linear algebraic equations with the Ritz parameters, by formulas (15) it is possible to calculate the displacement.

The strain and stress are determined by the expressions given in (13):

$$\sigma^{33}=0;\quad \sigma^{23}=\frac{E}{1+\nu}g^{\alpha\gamma}e_{\gamma 3};$$

$$\sigma^{\alpha\beta}=\frac{E}{1+\nu}\left[\frac{\nu}{1-\nu}g^{\alpha\beta}g^{\gamma\delta}+\frac{1}{2}\left(g^{\alpha\gamma}g^{\beta\delta}+g^{\alpha\delta}g^{\beta\gamma}\right)\right]\cdot e^{\gamma\delta};$$

$$2e_{\alpha\beta}=\mu_{\alpha}^{\nu}(Y_{\nu\beta}+z\Phi_{\nu\beta})+\mu_{\beta}^{\nu}(Y_{\nu\alpha}+z\Phi_{\nu\alpha});$$

$$2e_{\alpha 3}=\psi_{\alpha},\ e_{33}=0 \quad (25)$$

Here the metric tensor of the shell is equal:

$$g^{\alpha\beta}=(\mu^{-1})_{\nu}^{\alpha}(\mu^{-1})_{\alpha}^{\beta}\alpha^{\nu\lambda};$$

$$(\mu^{-1})_{\nu}^{\alpha}\approx\delta_{\nu}^{\alpha}+x^3 b_{\alpha}^{\nu}.$$

In the case of a thin shell $g^{\alpha\beta}=\alpha^{\nu\lambda}$.

To formulate the optimization problem, we require that σ_i stress intensity at any point of the cross-section do not exceed $[\sigma]$ allowable stresses:

$$\sigma_i \le [\sigma]. \qquad (26)$$

The stress intensity is determined by the formula:

$$\sigma_i = \frac{1}{\sqrt{2}}\sqrt{3\pi_2 - \pi_1} = \sqrt{\frac{3}{2}}\sqrt{S \cdot S},$$

where $\pi_1 = g_{ik}\sigma^{ik}$; $\pi_2 = g_{ik}\sigma^{ik} = g_{in}g_{km}\sigma^{nm}\sigma^{ik}$;

$$S^{ik} = \sigma^{ik} - g^{ik}\frac{1}{3}\pi_1.$$

In case of medium thickness shell, that corresponds to S.P. Timoshenko`s model, we have nonzero components.

$$\sigma^{11}, \sigma^{12}, \sigma^{13}, \sigma^{22}, \sigma^{33}.$$

As the target function we take the sensitivity of wound manometric spring, determined by the ratio of the rotation angle of the cross-section per unit θ length to the P internal pressure:

$$\tau_0 = \frac{Q}{P}. \qquad (27)$$

We consider the rotation tensor:

$$\omega_{ik} = \frac{1}{2}(U_{\alpha/3} - U_{3/\alpha}). \qquad (28)$$

Here U_i is the displacement vector in the metric of the shell body; the slash / denotes covariant differentiation in this metric.

The components of the rotation vector relative to the x^1 x^3 axes:

It is known that

$$U_{\alpha/2} = \mu_\alpha^v u_{v,2}, \qquad (29)$$

where $\mu_\alpha^v = \delta_\alpha^v - x^2 b_\alpha^v$ α=1,2;

$$u_{2,\alpha} = \bar{u}_{2,\alpha} + b_\alpha^v \bar{u}_v = -\theta_\alpha. \qquad (30)$$

Here U_v is the displacement vector in the metric of the middle surface of the shell:

$$u_2 = \omega.$$

Taking into account the kinematic model (4), from formulas (28–30), we can find the angle of rotation at the point of the shell body relative to x^3:

$$\theta \equiv \omega_{12} = \frac{1}{2}(\mu_1^v \bar{u}_{v,2} - \bar{u}_{2,1} - b_1^v \bar{u}_v) = \frac{1}{2}(\mu_1^v \bar{u}_{v,2} + \theta_1).$$

The rotation angle of tube section defined as the average value:

$$\theta = \frac{1}{2Lh}\int_{-h}^{h}\int_{\ell=0}^{L} \theta dx^2 dl,$$

where L is the length of the perimeter of the median line and dl is the element of the length of the section line which is defined as:

$$dl = \sqrt{a_{11}}dx^1.$$

We perform the averaging over the thickness:

$$\bar{\theta}(\ell) = \frac{1}{4h}\int_{-h}^{h}\left((\mu_1^v \bar{u}_{v,2} + \theta_1)dx^2 = \right.$$

$$= \frac{1}{4h}\int_{-h}^{h}\left(\mu_1^v(\theta_1 + \psi_1) + \theta_1\right)dx^2 =$$

$$= \frac{1}{4h}\left\{2h\left[(\delta_1^v(\theta_v + \psi_v) + \theta_1)\right\}\right. = \frac{1}{2}(\theta_1 + \psi_1 + \theta_1) =$$

$$= \theta_1 + \frac{1}{2}\psi_1.$$

Thus, the rotation angle per unit length of the tube is:

$$\theta = \frac{1}{2L}\int_{\ell}^{L}\bar{\theta}(l)dl.$$

We assume that the required optimization parameters are the thickness of the tube and the characteristic geometric dimensions that define the cross-sectional shape.

Thus, the optimization problem consists in finding such a set of parameters at which the target function (27) takes the largest value, and the conditions (26) are satisfied, and the constructive and technological restrictions on form and size of the cross-section of the tube.

The solution of the problem as a problem of nonlinear mathematical programming we will find using a hybrid adaptive method [12], high efficiency of which is confirmed by comparison with many known methods

Conclusions

1. In this article, mathematical model of the stress-strain state of wound tubular manometric springs is created. This model makes it possible to calculate the stress-strain state as accurately as possible at any point of research object, and therefore, at solving the problem of optimization the parameters of the object, do not go beyond the limits of the main constraint

$$\sigma_i \le [\sigma],$$

where σ_i is stress intensity in i point;

$[\sigma]$ –is working stress.

2. The form in which the required displacements, deformations and stresses are presented allows us to use a hybrid adaptive method to solve the problem of optimization of spring parameters, the high efficiency of which is confirmed by comparison with other methods.

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РОЗРАХУНОК ЧУТЛИВОСТІ КРУЧЕНИХ ТРУБЧАСТИХ МАНОМЕТРИЧНИХ ПРУЖИН СЕРЕДНЬОЇ ТОВЩИНИ МЕТОДОМ ТЕОРІЇ ОБОЛОНОК

О.С. Чубукін

У даній роботі викладено новий підхід до вирішення задачі розрахунку напружено-деформованого стану чутливого елемента манометричних і теплових приладів, який пропонується застосовувати в процесі його проектування.

Об'єктом дослідження є вита манометрична пружина.

Предметом дослідження є напружено-деформований стан цієї пружини з різними формами перерізів.

Метою дослідження є створення математичної моделі напружено-деформованого стану об'єкта, що дозволяє, в подальшому, вирішити завдання оптимізації його конструктивних параметрів в алгоритмі автоматичного розрахунку при автоматичному проектуванні пружин.

В результаті дослідження була створена модель напружено-деформованого стану кручених манометричних пружин, виконана постановка задачі оптимізації її параметрів з урахуванням конструктивних і технологічних обмежень та запропоновано метод розв'язання цієї задачі.

Область використання – авіаційні прилади, призначені для вимірювання тиску рідин і газів.

Ключові слова: *математична модель, напружено-деформований стан, скручена трубчаста манометрична пружина, оптимізація цільової функції.*

РАСЧЕТ ЧУСТВИТЕЛЬНОСТИ ВИТЫХ ТРУБЧАТЫХ МАНОМЕТРИЧЕСКИХ ПРУЖИН СРЕДНЕЙ ТОЛЩИНЫ МЕТОДОМ ТЕОРИИ ОБОЛОЧЕК

А.С. Чубукин

В данной работе изложен новый подход к решению задачи расчета напряженно-деформированного состояния чувствительного элемента манометрических и тепловых приборов, который предлагается применять в процессе его проектирования.

Объектом исследования является витая манометрическая пружина.

Предметом исследования является напряженно-деформированное состояние этой пружины с различными формами сечений.

Целью исследования является создание математической модели напряженно-деформированного состояния объекта, позволяющей, в дальнейшем, решить задачу оптимизации его конструктивных параметров в алгоритме автоматического расчета при автоматическом проектировании пружин.

В результате исследования была создана модель напряженно-деформированного состояния витых манометрических пружин, выполнена постановка задачи оптимизации ее параметров с учетом конструктивных и технологических ограничений и предложен метод решения этой задачи.

Область использования – авиационные приборы, предназначенные для измерения давления жидкостей и газов.

Ключевые слова: *математическая модель, напряженно-деформированное состояние, скрученная трубчатая манометрическая пружина, оптимизация целевой функции.*