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DIGRESSION ON THE RIGHT OFF-BOUND PROJECTOR OPTIMAL STRATEGY IN FOUR PROPS CONSTRUCTION BEING PRESSED UNCERTAINLY

There has been broached a problem of sharing the unit total cross-section square optimally on four props in support construction. The model of this is an antagonistic game with the kernel, maximizing a four-elemented set of special ratios. A theorem on specifying the projector optimal strategy in the case of underestimated uncertainties of pressing the props has been proved.

Keywords: *support construction, pressing uncertainly, unit-normed squeeze, high durability, low massiveness, cross-section square, removing uncertainties, antagonistic game, strict convexity, projector optimal strategy, under-estimated uncertainties, off-bound optimal strategy.*

Pre-discussing the problem generally

The world environment is natured so that there wants and needs are always bigger than availabilities. However, wants and needs are limited not only with availabilities, but also with contradicting among them-selves. In a way, when constructing something me-chanical or building, there high durability stands against low massiveness [1]. High durability is a need, and low massiveness may be as need as well as want, but simul-taneously they are not available in the sense of unattain-ability and discrepancy [1, 2]. Besides, any constructing process entails many uncertainties in evaluating parame-ters of the construction. Resolving them is an obvious neverending problem.

On the problem recent investigations and promulgations

There are works [1, 3, 4] on constructing processes and building mechanics, having calculations, estima-tions, expectations, usage, stiffness, bending and stress analysis, structural analysis, reliability calculation. Nev-ertheless, just starting from the simplest propping con-structions, there emerge uncertainties to be removed [5, 6] in a model, what will drive to a better relationship between durability and total weight of the construction. If considering the construction with four props of a definite shape (that actually is not of principle, as well as the scheme of these props location), then any unit-normed potential (nonpoint-before-evaluated) squeeze x_i on the i -th prop is

$$x_i \in [a_i;\, b_i] \subset (0;\, 1) \subset [0;\, 1] \tag{1}$$

by the conditions $\mu_R([a_i; b_i]) > 0 \quad \forall\, i = \overline{1,\, 4}$ and $\sum_{i=1}^{4} x_i = 1$. As an antisqueeze to x_i the i -th prop cross-section square y_i is unit-normed and

$$y_i \in [a_i;\, b_i] \subset (0;\, 1) \subset [0;\, 1] \tag{2}$$

by $\sum_{i=1}^{4} y_i = 1$. Having removed those $\{[a_i;\, b_i]\}_{i=1}^{4}$ – uncertainties will allow sharing the unit total cross-section square on four props optimally for guaranteeing the construction reliability within potential uncertain squeezes on it. For that the maximized ratios $\left\{\frac{x_i}{y_i^2}\right\}_{i=1}^{4}$ should be minimized [7]. And so, as a model of accom-plishing it, there was proposed an antagonistic game with the kernel

$$\begin{aligned} T(\mathbf{X},\, \mathbf{Y}) &= T(x_1,\, x_2,\, x_3;\, y_1,\, y_2,\, y_3) = \\ &= \max\left\{\{T_i(x_i,\, y_i)\}_{i=1}^{4}\right\} = \\ &= \max\left\{\frac{x_1}{y_1^2},\, \frac{x_2}{y_2^2},\, \frac{x_3}{y_3^2},\, \frac{1-x_1-x_2-x_3}{(1-y_1-y_2-y_3)^2}\right\} \end{aligned} \tag{3}$$

being defined on $\mathfrak{I}^6$ -hyperparallelepiped

$$\begin{aligned} \boldsymbol{X} \times \boldsymbol{Y} &= \prod_{i=1}^{2}[a_1;b_1]\times[a_2;b_2]\times[a_3;b_3] = \\ &= \prod_{i=1}^{2}\left(\prod_{d=1}^{3}[a_d;b_d]\right) \subset \prod_{j=1}^{6}(0;1) \subset \prod_{j=1}^{6}(0;1) \subset \mathfrak{I}^6 \end{aligned} \tag{4}$$

as a Cartesian product of $\mathfrak{I}^3$ -parallelepiped

$$\begin{aligned} \boldsymbol{X} &= [a_1;b_1]\times[a_2;b_2]\times[a_3;b_3] = \\ &= \prod_{d=1}^{3}[a_d;b_d] \subset \prod_{d=1}^{3}(0;1) \subset \prod_{d=1}^{3}(0;1) \subset \mathfrak{I}^3 \end{aligned} \tag{5}$$

of the first player pure strategies (casual circs, provok-ing those uncertainties)

$$\mathbf{X} = [x_1 \quad x_2 \quad x_3] \in [a_1;b_1]\times[a_2;b_2]\times[a_3;b_3] = \boldsymbol{X} \tag{6}$$

and of $\mathfrak{I}^3$ -parallelepiped

$$\boldsymbol{Y} = [a_1;b_1]\times[a_2;b_2]\times[a_3;b_3] = \\ = \prod_{d=1}^{3}[a_d;b_d] \subset \prod_{d=1}^{3}(0;1) \subset \prod_{d=1}^{3}(0;1) \subset \mathfrak{I}^3 \quad (7)$$

of the second player (the construction projector) pure strategies

$$\mathbf{Y} = [y_1 \quad y_2 \quad y_3] \in [a_1;b_1]\times[a_2;b_2]\times[a_3;b_3] = \boldsymbol{Y} . (8)$$

It is easy to see the game with the kernel (3) on the hyperparallelepiped (4) is conditionally strictly convex, and so the projector has a pure optimal strategy

$$\mathbf{Y}_* = [y_1^* \quad y_2^* \quad y_3^*] \in [a_1;b_1]\times[a_2;b_2]\times[a_3;b_3] = \mathbf{Y} \quad (9)$$

with giving a possibility to distribute the unit total cross-section square on four props as y_i^* on the i-th prop, $i = \overline{1,\ 4}$.

Sketching the intention and stating the tasks

There are no any difficulties in determining the regular optimal strategy (9) when the preliminary minimaxed-found [7, 8] components $\left\{y_d^*\right\}_{d=1}^{3}$ are within segments $\left\{[a_d;\ b_d]\right\}_{d=1}^{3}$. However, it always is occurring so that, say, three from the four squeezes on props had been underestimated and the right ends $\{b_d\}_{d=1}^{3}$ appeared to be less than expected. Thus the intention of the current paper lies in isolating such case with the underestimated right ends $\{b_d\}_{d=1}^{3}$. Then, firstly, it ought to be compared with the regularity case by the preliminary minimaxed-found components $\left\{y_d^*\right\}_{d=1}^{3}$ are within segments $\left\{[a_d;\ b_d]\right\}_{d=1}^{3}$. Secondly, it ought to be substantiated the determination of the strategy (9) within the underestimated right ends $\{b_d\}_{d=1}^{3}$ condition. Practically those ones are going to ensure the guaranteed optimum in unit cross-section square sharing on four props.

Within-bound (regular) projector optimal strategy

The projector optimal strategy (9) is found by the minimaxing the kernel (3) as the antagonistic game with the kernel (3) on the hyperparallelepiped (4) is conditionally strictly convex:

$$\min_{\mathbf{Y}\in\boldsymbol{Y}}\max_{\mathbf{X}\in\boldsymbol{X}} T(\mathbf{X},\mathbf{Y}) = \min_{\mathbf{Y}\in\boldsymbol{Y}}\max_{\mathbf{X}\in\boldsymbol{X}}\left\{\max\left\{\frac{x_1}{y_1},\frac{x_2}{y_2},\frac{x_3}{y_3},\frac{1-x_1-x_2-x_3}{(1-y_1-y_2-y_3)^2}\right\}\right\} =$$

$$= \min_{\mathbf{Y}\in\boldsymbol{Y}}\left(\max\left\{\max_{x_1\in[a_1;b_1]}\left\{\frac{x_1}{y_1^2}\right\}, \max_{x_2\in[a_2;b_2]}\left\{\frac{x_2}{y_2^2}\right\}, \max_{x_3\in[a_3;b_3]}\left\{\frac{x_3}{y_3^2}\right\}, \max_{\mathbf{X}\in\boldsymbol{X}}\left\{\frac{1-x_1-x_2-x_3}{(1-y_1-y_2-y_3)^2}\right\}\right\}\right) = \quad (10)$$

$$= \min_{\mathbf{Y}\in\boldsymbol{Y}}\left(\max\left\{\frac{b_1}{y_1},\frac{b_2}{y_2},\frac{b_3}{y_3},\frac{1-a_1-a_2-a_3}{(1-y_1-y_2-y_3)^2}\right\}\right).$$

The minimum of the hypersurface (10) as function of variable (8) on the parallelepiped (7) may particularly be reached at the point $[y_1 \quad y_2 \quad y_3] = [y_1^* \quad y_2^* \quad y_3^*]$, where the optimal game value v_* is

$$v_* = \frac{b_d}{\left(y_d^*\right)^2} = \frac{1-a_1-a_2-a_3}{\left(1-y_1^*-y_2^*-y_3^*\right)^2} \quad \forall\ d = \overline{1,\ 3}. \quad (11)$$

Then the equality (11) has the roots

$$y_d^* = \sqrt{\frac{b_d}{v_*}} \quad \forall\ d = \overline{1,\ 3}$$

and as

$$1 - \sum_{d=1}^{3} y_d^* = \frac{1}{\sqrt{v_*}}\sqrt{1-\sum_{d=1}^{3} a_d}$$

then

$$1 = \sum_{d=1}^{3}\sqrt{\frac{b_d}{v_*}} + \sqrt{\frac{1-a_1-a_2-a_3}{v_*}} = \\ = \frac{\sum_{d=1}^{3}\sqrt{b_d} + \sqrt{1-\sum_{d=1}^{3} a_d}}{\sqrt{v_*}}; \quad (12)$$

$$v_* = \left(\sum_{d=1}^{3}\sqrt{b_d} + \sqrt{1-\sum_{d=1}^{3} a_d}\right)^2. \quad (13)$$

The optimal game value (13) gives the regular [9] optimal strategy (9) with components

$$y_k^* = \frac{\sqrt{b_k}}{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}} = \\ = \frac{\sqrt{b_k}}{\sum_{d=1}^{3}\sqrt{b_d} + \sqrt{1-\sum_{d=1}^{3} a_d}},\ k = \overline{1,\ 3}. \quad (14)$$

The $\boldsymbol{Y}$ -within-bound projector optimal strategy (9) with components (14) requires that there would be

$$\frac{\sqrt{b_k}}{\sum_{d=1}^{3}\sqrt{b_d}+\sqrt{1-\sum_{d=1}^{3}a_d}} \in [a_k ; b_k] \quad \forall\ k=\overline{1,3} \tag{15}$$

simultaneously. But if squeezes on props had been underestimated and the right ends $\{b_d\}_{d=1}^{3}$ appeared to be less than expected, then

$$\frac{\sqrt{b_k}}{\sum_{d=1}^{3}\sqrt{b_d}+\sqrt{1-\sum_{d=1}^{3}a_d}} > b_k \quad \forall\ k=\overline{1,3} \tag{16}$$

and this is the case with the right off-bound projector optimal strategy.

Off-bound (completely nonregular) projector optimal strategy as corollary of (16)

As by (16) the straight minimaxing discourse with supposition (11) can't lead to the projector optimal strategy (9), then there should be searched a way to get to it properly. The following theorem opens the proved way.

Theorem. In the antagonistic game with the kernel (3) on the hyperparallelepiped (4) in the case of (16) there is the right off-bound projector optimal strategy

$$\mathbf{Y}_* = [b_1 \quad b_2 \quad b_3]. \tag{17}$$

Proof. The projector optimal strategy must belong to the parallelepiped (7) and its components are arguments of the minimum (10). As (11) is unreachable and

$$y_k^* < \frac{\sqrt{b_k}}{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}} =$$

$$= \frac{\sqrt{b_k}}{\sum_{d=1}^{3}\sqrt{b_d}+\sqrt{1-\sum_{d=1}^{3}a_d}} \quad \forall\ k=\overline{1,3} \tag{18}$$

then

$$\frac{b_k}{\left(y_k^*\right)^2} > \frac{1-a_1-a_2-a_3}{\left(1-y_1^*-y_2^*-y_3^*\right)^2} \quad \forall\ k=\overline{1,3} \tag{19}$$

whatever y_k^* is (because of the last formula left side denominator became less, and the right side denominator became greater). Thus the minimax (10) is

$$\min_{\mathbf{Y}\in\boldsymbol{Y}}\left(\max\left\{\frac{b_1}{y_1},\frac{b_2}{y_2},\frac{b_3}{y_3},\frac{1-a_1-a_2-a_3}{(1-y_1-y_2-y_3)^2}\right\}\right) =$$

$$= \min_{\mathbf{Y}\in\boldsymbol{Y}}\left(\max\left\{\frac{b_1}{y_1},\frac{b_2}{y_2},\frac{b_3}{y_3}\right\}\right). \tag{20}$$

But whatever the maximization within (20) result is, to minimize it the projector must use as great components as reachable. And they, generally speaking, can't equalize those three ratios, but

$$v_* = \max\left\{\frac{b_1}{\left(y_1^*\right)^2}, \frac{b_2}{\left(y_2^*\right)^2}, \frac{b_3}{\left(y_3^*\right)^2}\right\} =$$

$$= \max\left\{\frac{b_1}{(b_1)^2}, \frac{b_2}{(b_2)^2}, \frac{b_3}{(b_3)^2}\right\} =$$

$$= \max\left\{\frac{1}{b_1}, \frac{1}{b_2}, \frac{1}{b_3}\right\} \tag{21}$$

and the optimal game value (21) is reached at (17). The theorem has been proved.

Note that the optimal game (now nonstrictly convex) value (21) doesn't depend upon the left ends values $\{a_d\}_{d=1}^{3}$, whatever they are.

Consider an example.

If the $\{[a_d ; b_d]\}_{d=1}^{3}$ -uncertainties segments are

$$\{[a_d ; b_d]\}_{d=1}^{3} =$$

$$= \{[0.11; 0.21], [0.12; 0.16], [0.1; 0.17]\} \tag{22}$$

then

$$\frac{\sqrt{b_1}}{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}} =$$

$$= \frac{\sqrt{0.21}}{\sqrt{0.21}+\sqrt{0.16}+\sqrt{0.17}+\sqrt{1-0.33}} \approx$$

$$\approx 0.219356 > b_1 = 0.21,$$

$$\frac{\sqrt{b_2}}{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}} =$$

$$= \frac{\sqrt{0.16}}{\sqrt{0.21}+\sqrt{0.16}+\sqrt{0.17}+\sqrt{1-0.33}} \approx$$

$$\approx 0.1914697 > b_2 = 0.16,$$

$$\frac{\sqrt{b_3}}{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}} =$$

$$= \frac{\sqrt{0.17}}{\sqrt{0.21}+\sqrt{0.16}+\sqrt{0.17}+\sqrt{1-0.33}} \approx$$

$$\approx 0.197362 > b_3 = 0.17,$$

and so

$$\mathbf{Y}_* = [0.21 \quad 0.16 \quad 0.17]$$

at once due to the proved theorem.

The optimal game value (21) here

$$v_* = \max\left\{\frac{1}{0.21}, \frac{1}{0.16}, \frac{1}{0.17}\right\} = \frac{1}{0.16} = 6.25 \tag{23}$$

is out of practical interest. Though, if trying to obtain the regular optimal strategy (9) then, instancing, new uncertainties segments

$$\left\{\left[a_d;\ b_d\right]\right\}_{d=1}^{3} =$$

$$= \left\{\left[0.11;\ 0.24\right],\ \left[0.12;\ 0.21\right],\ \left[0.1;\ 0.22\right]\right\} \qquad (24)$$

with the increased right ends give the optimal game value (13)

$$v_* = \left(\sqrt{0.24} + \sqrt{0.21} + \sqrt{0.22} + \sqrt{1 - 0.33}\right)^2 \approx$$

$$\approx 4.998499 \qquad (25)$$

appearing to be decreased in comparison to (23).

The investigation conclusion and outlooks for furthering

To prop optimally the construction or platform within conditions of pressing it uncertainly the projector should ensure the minimum of maximal disbalance in squeezes on props, thus smoothing the total squeeze. The right off-bound (completely nonregular) projector optimal strategy as corollary of (16) may speak that three from the four squeezes on props was underestimated and for getting the regular optimal strategy (9) those right ends $\left\{b_d\right\}_{d=1}^{3}$ ought to be evaluated greater. This probably may decrease the optimal game value as the total overpress effect, like it just has been demonstrated in the example above. Thus regularizing the right off-bound projector optimal strategy (17) is a clear mattered further investigation.

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ВІДСТУП ДО ПРАВОЇ ПОЗАГРАНИЧНОЇ ОПТИМАЛЬНОЇ СТРАТЕГІЇ ПРОЕКТУВАЛЬНИКА У КОНСТРУКЦІЇ З ЧОТИРМА ОПОРАМИ, НА КОТРІ ДІЄ НЕВИЗНАЧЕНИЙ ТИСК

В.В. Романюк

Порушено задачу оптимального розділення одиничної загальної площі поперечного перерізу на чотири опори в опорній конструкції. Моделлю цього є антагоністична гра з ядром, що максимізує чотирьохелементну множину зі спеціальних співвідношень. Доведено теорему щодо встановлення оптимальної стратегії проектувальника у випадку недооцінених невизначеностей стиснення опор.

***Ключові слова:** опорна конструкція, невизначене стискування, нормований до одиниці стиск, висока зносостійкість, низька масивність, площа поперечного перерізу, усунення невизначеностей, антагоністична гра, строга опуклість, оптимальна стратегія проектувальника, недооцінені невизначеності, позагранична оптимальна стратегія.*

ОТСТУПЛЕНИЕ К ПРАВОЙ ВНЕГРАНИЧНОЙ ОПТИМАЛЬНОЙ СТРАТЕГИИ ПРОЕКТИРОВЩИКА В КОНСТРУКЦИИ С ЧЕТЫРЬМЯ ОПОРАМИ, НА КОТОРЫЕ ДЕЙСТВУЕТ НЕОПРЕДЕЛЁННОЕ ДАВЛЕНИЕ

В.В. Романюк

Затронута задача оптимального разделения единичной общей площади поперечного сечения на четыре опоры в опорной конструкции. Моделью этого является антагонистическая игра с ядром, максимизирующим четырёхэлементное множество специальных соотношений. Доказано теорему по установлению оптимальной стратегии проектировщика в случае недооцененных неопределённостей сжатия опор.

***Ключевые слова:** опорная конструкция, неопределённое сжатие, нормированное к единице сжатие, высокая износоустойчивость, низкая массивность, площадь поперечного сечения, устранение неопределённостей, антагонистическая игра, строгая выпуклость, оптимальная стратегия проектировщика, недооцененные неопределённости, внеграничная оптимальная стратегия.*