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Y.P. Machekhin

Kharkov National University of Radio Electronics, Kharkov, Ukraine

MEASUREMENT SCIENCE FOUNDATIONS

Foundation of modern measurement science is principles of classical mechanics. They are included to some international metrological documents. The paper shows how new fundamental results in theory of nonlinear dynamic systems we can to apply in measurement science. General idea investigations of paper – realize influence of strange attractor on uncertainty measurement. It is discussed in paper how measurement science can be corrected in nonlinear dynamic systems.

Keywords: *nonlinear dynamic systems, strange attractor, uncertainty.*

1. Introduction

All methods for analysis of measurement results [1] based on physical models and correspondent mathematical methods. In particular, physical models of modern measurement science, founded on experimentally established facts of classical mechanics [2].

Main condition, which always holds in classical mechanics - initial state of the mechanical system uniquely determines its movement through space and time. As a mathematic basis, are using ordinary differential equations, which solutions described conditions of real physical systems. From ideas of classical mechanics, follow two general conditions. First, the measurand has a unique value [1]. Principles of Newton-Laplace determinism according to which the behaviour of the system in time and space described by ordinary differential equations. Second, measurement precision is determined initial conditions of the measurement experiments.

In dynamic systems may be not only regular motions but chaotic, non-regular behavior too. This type of system behavior are describing by a difference equations and well known in literature [3-6]. One type of solutions describes deterministic periodical behavior in time while other type of solutions describes chaotic non-regular behavior.

Modern development of measurement science connected with measurement tasks solutions in non linear dynamic systems, for example, in medicine, economy and social area.

In this report, we present results of investigation mathematical and physical foundations of non-linear dynamical system and its influence on measurement results. We discuss how to take into account strange attractor as a inside fluctuations in systems, based on mathematical description of chaotic behavior in real systems.

Fundamental condition which is underlaid measurement science consists in connection of the state of the dynamic system (motion in space and in time) from initial conditions.

Physically the initial condition of the dynamical system is known in final area of phase space.

Therefore a uncertainty, in which the value of measurand is set, is determined through the uncertainty of task of initial conditions of measuring experiment.

On another hand, following a theorem about local existence and unicity of decision of the system of usual differential equations [7], a physical value, described these differential equations, has the unique value, determined initial conditions.

If the system is steady, its conduct is predictable and, consequently, can be drawn on the results of measurings as objective description of the system on time of prediction domains.

Those, which in phase space are described as the steady attracting special points or steady, attracting trajectories, behave to the stable states (simple attractors [7]).

Mathematically the stable states in phase space are described as a knot or focus. The estimation of properties of motion of the dynamic system near-by which is related to the roots of characteristic equation [7].

Well known methods of analysis measurement results used in the dynamic systems only for those real states which in phase space correspond simple attractors, i.e. for those situations which the subzero real parts of roots of characteristic equation.

By analogical appearance, in the dynamic system the state of which is described a steady trajectory (for example, limit cycle), measurement are executed in area of attraction of trajectory in phase space.

As early as the middle of the last century, picture of chaos, as about an irregular casual process in the physical systems associated with the condition of excitation of very plenty of degrees of freedom. Exactly this model always was in basis of methods of estimation of error and uncertainty of measurement. It is necessary to take into account that correctness of methods of results estimation is based on stationary and ergodic casual processes.

Nevertheless, yet the first fundamental researches [3 – 5] of mathematical and physical basis of casual processes resulted in interesting results. To explaining, that in the dynamic systems for providing of statistical equilibrium «stirring» properties which consist in dependence trajectory of motion of the dynamic system in phase space from initial conditions must be realized.

A small rejection in initial conditions results in the substantial difference of possible trajectories of motion. Thus, first it was possibility of influence of conduct of the dynamic system is well-proven on properties of casual processes.

Since 60th of past century it was strictly well-proven as a result of numerous (and above all things mathematical) researches, that a casual conduct can be registered in the fully determined systems with the low degrees of freedom.

Mechanism, which is responsible for the casual conduct of the determined system, consists in local instability of the system which together with dissipation results in the stable state of this system in the final volume of phase space.

This state is characterized the difficult non regular mode of conduct in the limited area of phase space which names-this by the determined (dynamic) chaos [8].

Nature of dynamic chaos is investigated in dissipative [9] and conservative [10] systems. The principle difference of the dissipative systems from conservative consists in existence of attracting sets - strange attractors [11].

A strange attractor is a set in phase space to which trajectories are attracted from the vinicity of this set, but on a set motion has unsteady character exponentially

The purpose of the real work was further development of measurement science [12 – 15], for providing of correctness of measurement in the conditions of strange attractor.

It is in-process rotined that the methods of estimation of uncertainty of measurement can be based on the high-quality theory of differential equations and used for the analysis of measurement results in the nonlinear dynamic dissipative systems.

2. Measurement basis in Nonlinear dynamical systems

Examining the dynamic system as purpose of measurement task, follows as equation of measurement, this binds a measurand to other parameters, to use the decision of mathematical model of the dynamic system.

Dynamic system the parameters of which it is planned to measure, is usually described by n – dimention system of differential equations (1), which has an analytical decision

$$\frac{\partial}{\partial t}\vec{X} = \vec{f}(\vec{X}), \quad (1)$$

where: $\vec{X} = (X_1, X_2 ... X_n)$ set of dynamic variables, characterizing the state of the dynamic system; $\vec{f} = (f_1, f_2 ... f_n)$ vectorial function, usually it is smooth [4] and certain in part of phase space.

A measurement task in the dynamic system makes sense only in the case when there are the stationary, steady and attracting states.

Mathematical description of the state of the dynamic system near-by the special points, carried out by the Lyapunov method [7].

A feature of analysis of measurement results in the dynamic systems, being near-by the special points related to the law on which a return of the system is in the stable state.

Near-by a stationary attracting point, it is necessary to take into account indignations systems, conditioned an external noise.

Transforming variables in equation (1), it is possible near-by a steady point to rewrite equation which will describe motion of the system near attracting point and which can be used as equation of measurement. The size of rejection η is for this purpose entered, on condition that $\eta = X\text{-}X^{(0)} << X^{(0)}$. We will take into account that on the conduct of the dynamic system an additive external noise influences near-by a point $- X^{(0)}$.

The most simple equation, describing the dynamic system near-by a steady point in phase space, is linear equation of Langeven [16]

$$\frac{\partial}{\partial t}\eta = -\lambda\eta + \varphi(t), \quad (2)$$

with initial conditions $\eta(0) = A_0$ and $\varphi(0) = \Phi_0$. Sign of index « $-\lambda$ » demonstrates the conduct of the system near-by a steady point. In the conditions of absence of noises the change of size $\eta(t)$ is described expression $\eta(t) = A_0 \exp(-\lambda t)$.

Equation, describing the conduct of the dynamic system, looks like $X = X^{(0)} + A_0 \exp(-\lambda t)$. For time $t = 1/\lambda$ the system from the perturbative state goes back into the stable state $X = X^{(0)}$.

Fluctuations of size η under the action of random forces $\varphi(t)$ are described expression

$$\eta(t)=\int_0^t d\tau\varphi(\tau)\exp\{-\lambda(t-\tau)\}\,. \qquad (3)$$

Clearly, that the pattern of temporal behaviour of process $\eta(t)$ will be also determined speed of returning in the stable state, i.e. by a size λ.

If to examine casual forces as a sequence delta functions, destroying the system from the state of steady equilibrium, expression for $\eta(t)$ simplified

$$\begin{aligned}\eta(t)&=\int_0^t d\tau\sum_{m=1}\delta(\tau-\tau_m)\exp\{-\lambda(t-\tau)\}=\\&=\sum_{m=1}\exp\{-\lambda(t-\tau_m)\}.\end{aligned} \qquad (4)$$

Thus, if the task of measuring of size $X^{(0)}$ is put, on results measuring of size X it is possible to set the area (great number) of values which can be added, to the probed size.

Statistical variation, measured values of size X, there is a set X_i at $1<i<N$. Mean value of the measured sizes

$$\begin{aligned}\langle X\rangle&=\frac{1}{N}\sum_{i=1}^{N}X_i=\frac{1}{N}\sum_{i=1}^{N}\eta_i+X^{(0)};\\ \eta_i&=\eta(t_i).\end{aligned} \qquad (5)$$

The mean value of measurands differs from the value of the stable state on the average of deviation from the stable state

The mean value of size of deviation from a stationary value in times of supervision of T is determined

$$\begin{aligned}\langle\eta\rangle&=\frac{1}{T}\int_0^{\text{Ò}} dt\eta(t)=\\ &-\frac{1}{T}\int_0^T dt\int_0^t d\tau\varphi(\tau)\exp\{-\lambda(t-\tau)\}.\end{aligned} \qquad (6)$$

Thus standard uncertainty of type A is determined

$$u=\sqrt{\frac{1}{T}\int_0^T dt(\eta(t)-\langle\eta\rangle)^2}\,. \qquad (7)$$

3. Measurement basis in the field strange attractor

Rising of measurement task is in the dynamic system, being in a state of the determined chaos has a row of features.

At first, on results preliminary researches existence of the mode of strange attractor is set [17 – 21].

Secondly, all fixed values of measurand correspond the real state of the system.

Thirdly, under the uncertainty vagueness of measuring result the size of attracting great number of attractor is understood in phase space.

Experimental researches of the chaotic modes of the dynamic system are carried out by treatment of temporal rows of results of supervisions, registered for period of time much greater what characteristic time of strange attractor.

The methods of analysis of temporal rows, in-use for the study of chaotic motion [8], allow determining such descriptions as a fractal dimension, indexes of Lyapunov, entropy and investment dimension [11].

A temporal row of results of supervisions is a sequence of values $x(t)$, got through a temporal interval τ, $t_i=t_0+(i-1)\tau$, $x_i=x(t_i)$, $i=1,...,N$. For research of structure of strange attractors on the temporal rows of supervisions of F. Takens [22] mathematically developed and grounded a vehicle on the basis of application of vectors of delay.

This method uses expression for a cross-correlation integral

$$C_m(\varepsilon)=\lim_{N\to\infty}\sum_{i,j=1}^{N}\frac{1}{N^2}\Theta\left(\varepsilon-\left|y_i^{(m)}-y_j^{(m)}\right|\right), \qquad (8)$$

where Θ − Hevisaid function; ε − distance between the points of set on an attractor; $\left|y_i^{(m)}-y_j^{(m)}\right|$ − modul of distance between two points i and j set, formed on principle

$$\left|y_i^{(m)}-y_j^{(m)}\right|=\sqrt{\sum_{n=0}^{m-1}(x_{i+n}-x_{j+n})^2}\,.$$

Elements x_i are the results of initial supervisions on which a size m is formed in phase space of dimension $y^{(m)}$ ($y_i^{(m)}=\{x_i,x_{i+1},...x_{i+m-1}\}$).

Analysis of unidimensional temporal row, got as a result of successive in time registration of values probably on a strange attractor with the use (8) allows to define the dimension of space of investment $-M$, fractal dimension $-\ D$.

For strange attractors there always is such maximal value ε exceeding of which does not result in the increase of value of cross-correlation integral $C(\varepsilon)$ [23].

The use of cross-correlation integral allows setting existence of the mode of strange attractor; therefore there must the first step be a calculation of cross-correlation integral at implementation of measurings and estimation of uncertainty.

It is important for a measurement experiment, that the dynamics of conduct of the probed system is such, that if in initial moment of time ($t=0$) the uncertainty of position of phase point of the system is characterized Ω_ε, through the protracted enough interval of time, size of area in which a phase point of the system can be and move, increased to the sizes of strange attractor.

To carry out the estimation of size of area as it applies to the looked after parameter it is necessary to take advantage of temporal rows of results of supervisions.

In area of strange attractor the casual conduct of the probed size is conditioned the unsteady dynamics of conduct of the system, therefore for the estimation of uncertainty of measurement it is necessary to be based on a mathematical model which describes the trajectory of motion.

As, to build an analytical model for the nonlinear dynamic system, in most cases, practically, it is impossible, therefore for the estimation of vagueness of results of measurings can be used, or results of numeral design or results x_i of experimental supervisions.

If at the calculation of the extended uncertainty U the interval of credible values of measurand is set, in the case of strange attractor the estimation of uncertainty u is executed on the maximal scope of the looked after sizes

$$u = \max_{i,j=1...N} \left| x_i - x_j \right|. \quad (9)$$

Actually, the maximal diameter of unidimensional great number of results of supervisions can be examined as a basic estimation of uncertainty of measurement.

4. Conclusion

Examining from single positions the methods of analysis of measuring results in the linear and nonlinear dynamic systems, the features of influence of conduct of the dynamic systems were in-process exposed on the results of measurings.

A main feature, got results, consists in that with their help it is possible correctly to conduct the analysis of results of measurings in any dynamic systems of being both in a state of simple and strange attractors.

If the stable state is described a maximum cycle or steady focus, the estimation of results is carried out taking into account attracting properties of the stable states.

In that case, when the stable state is a strange attractor, the analysis of results of measuring experiment must be executed taking into account the sizes of attracting great number of attractor.

At measuring of one of parameters of the dynamic system, a vagueness is determined in size projections of section of set on the axis of the probed parameter.

Advantage of similar estimation of results of measurings consists in that is set interval of values which the probed variable accepts casual appearance

In dynamic systems such, as biological, medical and economic interests the interval of possible values which the probed size can accept in certain terms.

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Рецензент: д-р техн. наук, проф. І.П. Захаров, Харківський національний університет радіоелектроніки, Харків, Україна.

ОСНОВИ ВИМІРЮВАЛЬНОЇ НАУКИ

Ю.П. Мачехін

Основою сучасної вимірювальної науки є принципи класичної механіки. Вони включені в деякі міжнародні метрологічні документи. У роботі показано, як нові фундаментальні результати в теорії нелінійних динамічних систем можна застосовувати у вимірювальній науці. Генеральна ідея досліджень статті − усвідомлення впливу дивного аттрактора на невизначенысть вимірювань. У статті обговорюється, як корегувати вимірювальну науку в нелінійних динамічних системах.

***Ключові слова**: нелінійні динамічні системи, дивний аттрактор, невизначеність.*

ОСНОВЫ ИЗМЕРИТЕЛЬНОЙ НАУКИ

Ю.П. Мачехин

Основой современной измерительной науки являются принципы классической механики. Они включены в некоторые международные метрологические документы. В работе показано, как новые фундаментальные результаты в теории нелинейных динамических систем можно применять в измерительной науке. Генеральная идея исследований статьи − осознание влияния странного аттрактора на неопределенность измерений. В статье обсуждается, как корректировать измерительную науку в нелинейных динамических системах.

***Ключевые слова:** нелинейные динамические системы, странный аттрактор, неопределенность.*