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MODELING IN MEASUREMENT SCIENCE: FROM THE PRESCRIPTIVE MODEL TO THE DESCRIPTIVE MODEL FOR THE PRACTICAL IMPLEMENTATION OF THE FORMER

In measurement science the object of a measurement is called "measurand". It is defined in VIM:2012 clause 2.3 as "quantity intended to be measured" (but in GUM:1995, B.2.9 it is defined instead according to VIM:1993 clause 2.6 "particular quantity subject to measurement". In VIM:2012, NOTE 1 to clause 2.3 specifies "the specification of a measurand requires ... description [i.e., a model] of the state of the phenomenon, body, or substance". The concept of measurand should be shared by the relevant Community, because the same measurand is supposed to be the object of replicated measures that must be comparable, i.e., it should be recognised as a quantity having a current recognisable meaning for the community. In the dialect of the science philosophers, this means that it should be projected into a "social framework". Also in the scientific frame this means that the measurand model must be of the "prescriptive" type, meaning "giving directions or injunctions"— not always meaning "physical model". The design of an experiment (DoE) must start from this initial conceptual model of the measurand, "socially shared", not from the building up of the descriptive model (often called the "experimental model") of the measuring system, which is specific to each measurement arrangement. Also each measuring system must be modelled, based on the specific solutions that are chosen in order to implement the prescription in each experiment. The initial conceptual model, being independent on any specific experimental implementation, is clearly a highly idealised one. It does not even allow appreciating the experimental difficulties and compromise (which are graduated depending on the target uncertainty). They arise from three categories of sources indicated in the VIM:2012: 1) the phenomenon, body, or substance; 2) the measuring system; 3) the conditions under which the measurement is carried out. A corresponding model must describe the measurement conditions (often called "physical", "experimental" or "observation" model—here it does not necessarily correspond to any of them). The paper discusses the non-simple roadmap bringing from the prescriptive model to the descriptive one.

Keywords: *measurand, model of the measurand, prescriptive model, descriptive model, measurement conditions.*

Introduction

In measurement science the object of a measurement is called "measurand". It is defined in VIM:2012 [1] clause 2.3 as "quantity intended to be measured" (but in GUM:1995, [2] B.2.9 it is defined instead according to VIM:1993 [3] clause 2.6 "particular quantity subject to measurement".

In VIM:2012 [1], NOTE 1 to clause 2.3, specifies "the specification of a measurand requires … description [i.e., a model] of the state of the phenomenon, body, or substance …", and in NOTE 3 is added "the measurement, including the measuring system and the conditions under which the measurement is carried out, might change the phenomenon, body, or substance such that the quantity being measured may differ from the measurand as defined. In this case, adequate correction is necessary" (emphasis added: notice the difference to Note 3 of the above clause 2.17). The term "correction" in VIM:2012 clause 2.53 is "compensation for an estimated systematic effect" – while the term "bias", defined in clause 2.18, is "estimate of a systematic measurement error". Thus, systematic error/effect means that the "quantity being measured" (in a specific case) may not be the intended one: consequently the correct understanding of the errors/effects is intrinsic in the construction of the model.

The concept of measurand should be shared by the relevant Community, because the same measurand is supposed to be the object of replicated measurements that must be comparable, i.e., it should be recognised as a quantity having a current recognisable meaning for the community. In the dialect of the science philosophers, this means that it should be projected into a "social framework". [5] Also in the scientific frame this means that the measurand model must be of the "prescriptive" type, meaning "giving directions or injunctions" [6] – which does not always mean "physical model".

The design of an experiment (DoE) must start from this initial conceptual model of the measurand, "socially shared", not from the building up of the descriptive model (often called the "experimental model") of the measuring system, which is specific to each measurement arrangement. Also each measuring system must be modelled, based on the specific solutions that are chosen in order to implement the prescription in each experiment.

The initial conceptual model, being independent on any specific experimental implementation, is clearly a highly idealised one. The paper discusses the non-simple roadmap bringing from the prescriptive model to the descriptive one.

Exposition of basic material

The measurand prescriptive model

In the following the cause-effect diagram (or Ishikawa diagram [7]) will be used. It does not require an explicit formulation of the functional relationships, but indicates the cause-to-effect flow of information on which the relationships are based. It does not include the representation of the associated uncertanties.

In that type of representation, fig. 1 shows a generic model for the *prescriptive model*, where the quantities are linked by implicit functional relationships schematically indicated. The superscript ⊖ (borrowed from chemical-physics where it means "ideal state") indicates that the variables in the model have the meaning required by the prescription, i.e., be "socially shared"; it also avoids to confound that model with the measurement model, that is instead specific of any single arrangement. $^{\ominus}Y$ indicates the "intended quantity", the measurand.

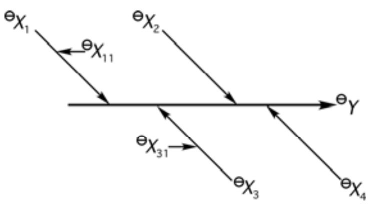

Fig. 1. Cause-effect diagram of a generic *prescriptive model* of the measurand. The subordinate lines show (with arrows added only for clarity) the direction of the flow of the input-quantities influence on the output quantity Y, irrespective to the direction of their slope. There are input quantities directly influencing $^{\ominus}Y$ (single subscript), others subordinated to other influence quantities (double subscript); further levels are possible, according to the complexity of $f(^{\ominus}X_i)$. For the meaning of the superscript ⊖ see text

All $^{\ominus}X$ corresponds to the expected value that the measurement should assign and that will be affected by a variance (except when stipulated), resulting in an expected value and variance of the measurand $^{\ominus}Y$.

Also, each measuring system must be modeled, based on the specific solutions that are chosen in order to implement the prescription. The initial conceptual model, being independent on any specific experimental implementation, does not even allow appreciating the experimental difficulties and compromise (which are graduated depending on the target uncertainty). They arise from three categories of sources indicated in the VIM:2012:

1) the phenomenon, body, or substance;
2) the measuring system;
3) the conditions under which the measurement is carried out.

A corresponding, but different, model must describe the actual measurement conditions (often called "physical", "experimental" or "observation" model – here not necessarily corresponding to any of them).

A generic descriptive model of a specific measurement system

An *generic actual model* for a specific measuring system is the one depicted in fig. 2, to be compared with the prescriptive model in fig. 1.

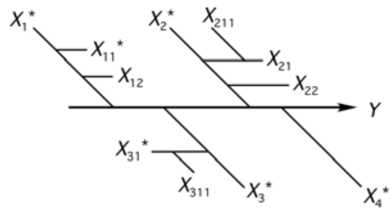

Fig. 2. *Generic model* for a *specific* measuring system: symbols has no reference to fig. 1, see text for their meaning

(Notice that $X = E(X) + {}^{0}X$ (zero-mean X))

The following differences with respect to fig. 1 apply:

- Y is *not* the measurand.
- X_i* are "localized" X_i
- X_{ij}, X_{ijk} are 'intricate' functional expressions to match the experimental complexity. ("Localized" means the quantity at a specific location of the experimental setup).

The functional expressions of the X_{ij} and X_{ijk} contain different quantities, or the quantities X_i for different locations of the measuring setup. When a quantity for any X_{ij} or X_{ijk} is Y, an iterative procedure has then to be used to compute Y.

The result of a measurement, Y, (VIM [1] clause 2.9: "result of measurement set of quantity values being attributed to a measurand together with any other available relevant information"; GUM B.2.11 [2]: "value attributed to a measurand, obtained by measurement") must be specified if it is: the indication, the uncorrected result or, the corrected result.

What is being measured is almost never the aimed measurand, i.e., the system is not in its "reference" condition (i.e. it does *not* match the prescriptive state indicated with ⊖).

However, all quantities should have been assigned a reference condition, in order to allow determining the out-of-reference condition, i.e. what is called the "measurement bias" (VIM clause 2.18 [1]: "estimate of a systematic measurement error"; GUM 3.2.3 [2]: "systematic error, often termed bias") In the VIM and the

GUM (and in most literature) this requirement is interpreted as shown in fig. 3. It is like fig. 2, changed by labelling "bias", B, caused by systematic effects, all quantities X exceeding those in the prescriptive model.

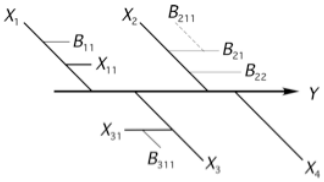

Fig. 3. Generic model for a *specific measuring system with bias indication*, according to GUM and VIM [1–2]

(Notice that $X = E(X) + {}^0X$; $B = E(B) + {}^0B$ (zero-mean X or B), where most often $E(B) \neq 0$)

On the contrary, as shown in fig. 4, the X must be "localized" (indicated by *), and, must be identified when they already are in their reference condition (⊖).

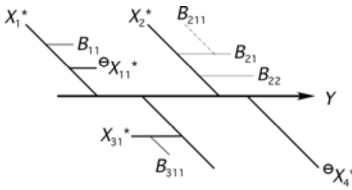

Fig. 4. Same as fig. 3 *with bias indication*, but with also indication of "localization" and of reference state

The figures picture "bias" as the *out-of-reference condition*. When a factor, B = (1 + B*); when an addend, B = B*, where B* is the out-of-reference expression, usually having E(B*) ≠ 0.

The final descriptive model

Its basic feature is to *compensate for the deviations from the ideal state*, in order to obtain results comparable with other measurements of the same measurand, a necessary condition in science.

The B's should be called "input quantities" too, affecting the quantities of the prescriptive model after their localization – when needed.

Therefore, all input quantities can be affected by bias, because all may happen to be measured when being in an out-of-reference condition.

The B's are no more "systematic" then the X's. They are as important as the X's, as far they "influence" the value of Y by exceeding the target uncertainty; otherwise they would not appear in the model.

The B are not "errors" nor responsible for errors. Should happen that a $E(B_i*) = 0$, this is not equivalent to omit it as an influence quantity, since ${}^0B_i$* stands.

Out-of-reference does mean a functional dependence of each specific quantity on further quantities, consisting in general, of sub-equations, if in analytical form, or in tables of values, or in differences of values, affected by an uncertainty with Type A/Type B components. One state for each independent variable needs be taken as the reference, to which a reference state of each dependent variable corresponds, and, consequently, the output quantity Y can be said to be in its reference state, $^\ominus$Y.

None of the models illustrated so far is a "measurement model" according to the VIM and GUM definitions – which are too generic and the latter with no associated "concept diagram".

So far, the model reported in fig. 4 looks the closest to the prescriptive one, but it is not appropriate yet for comparison with the descriptive model of the measurand, $^\ominus$Y. In order to transform it to become comparable with the measurand descriptive model for the specific experiment, it must be modified *in order to represent the reference state of each quantity*.

The key issue at this last step is that there is a *hierarchy* among the influence quantities, as evidenced in fig. 5: the operation of computing the value of each quantity at the reference state proceeds from the lower hierarchy level (triple subscript in the figure) to the top level (single subscript) – i.e., the level of the quantities appearing in the localised prescriptive model of fig. 4.

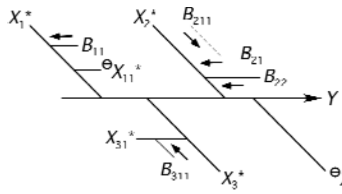

Fig. 5. Same as fig. 3 *with bias indication*, but also with indication of the hierarchical order to follow for normalisation

The final model is shown in fig. 6, where there is no further need to use, for any influence quantity, a symbol other than *X*. In it, the input quantities are:

$$X_i^*,\ X_{ij}^* \Leftarrow B_{ij}^*,\ X_{ijk}^* \Leftarrow B_{ijk}^*$$

(memo: as a factor B = (1 + B*), as an addend B = B*).

Normalisation is not due only for the quantities that are measured at their respective reference states (⊖).

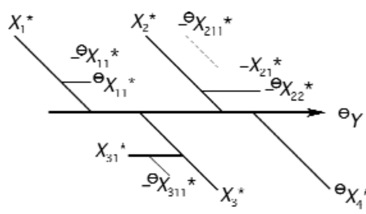

Fig. 6. The **final model** for the specific measuring system compatible with the measurand. Notice the minus signs that ensure "correction" to the reference state

The normalisation operation is today called "correction", i.e., "compensation for systematic effect / error".

However, it is instead shown that:

– the rational is to match the *reference condition* of the measurand expressed by the common prescriptive model;

– the aim of the specific measurement is the find a result for the measured quantity, Y, but considered as a "socially shared" quantity;

– all quantities in the final model of fig. 6 are *not* the same of the initial model, but can be related to it;

– Y can be related to $^{\ominus}$Y.

Conclusions

The model in fig. 6 brings to the correct measurement result that can be compared with replicated measurements of the same measurand. Notice that none of its quantities is that of the initial prescriptive model in fig. 1.

In fig. 7 the "concept diagram" indicated in VIM [1] for "measured quantity value" is compared with the concept diagrams resulting for the prescriptive and the descriptive models, respectively, in this paper.

The VIM model starts here from the "input quantities", defined as "2.50 input quantity in a measurement model (input quantity): quantity that must be measured, or a quantity, the value of which can be otherwise obtained, in order to calculate a measured quantity value of a measurand"; in GUM [2] (based on VIM:1993 [3]) as "4.1.2 The input quantities X_1, X_2, ..., X_N" are those on "which the output quantity Y depends"; they "may themselves be viewed as measurands and may themselves depend on other quantities, including corrections and correction factors for systematic effects". Finally, "Indications, corrections and influence quantities can be input quantities in a measurement model" (VIM 2.50, NOTE 2) [1], where:

"indication" (4.1 "quantity value provided by a measuring instrument or a measuring system", with "NOTE 2 An indication and a corresponding value of the quantity being measured are not necessarily values of quantities of the same kind";

"correction" (2.53 "compensation for an estimated systematic effect");

"influence quantity" (2.52 "quantity that, in a direct measurement, does not affect the quantity that is actually measured, but affects the relation between the indication and the measurement result"),

are all components of the measurement model related to the aimed measurand.

The GUM considers "influence quantity" as "B.2.10 quantity that is not the measurand but that affects the result of the measurement", [2] and "corrections" as affecting input quantities, so being implicit in the measurement equation $Y = f(X_1, X_2, ..., X_N)$, where Y is the measurand.

The VIM [1] notes in 2.52: "NOTE 2 In the GUM, the concept 'influence quantity' is defined as in the second edition of the VIM, covering not only the quantities affecting the measuring system, as in the definition above, but also those quantities that affect the quantities actually measured. Also, in the GUM this concept is not restricted to direct measurements". This clarifies that for the VIM "influence quantity" does *not* include any of the influence factors in *indirect* measurements—the most common type of measurement.

In this paper, instead, a conceptual "prescriptive model" is first set for the measurand, which is the model of the measurand at its *reference [ideal] state (target model)*. The input quantities, *all* named *influence quantities*, shown in fig. 2, are neither of the VIM or the GUM types since they apply to all influence factors and do *not* include the corrections, so they are the *uncorrected quantities* when in a out-of-reference condition. Should the measurement procedure allow reproducing the measurement result for the reference state, this model would also be that of the measurement model – very unlikely – and the "output quantities" would directly be the "social" one, in the sense that they can be shared or compounded with replicated measurements of the same or of other authors.

In most instances, instead, the measurement procedure depends on further quantities significantly influencing the measurement result, i.e. the system is in out-of-reference conditions: all are named "influence quantities" without the need for the VIM and GUM distinctions. In fact, should one abandon the concept of "true value", also the "systematic error" and "correction" terms should be abandoned.

These quantities are included in the "measurement model" shown in fig. 6, which is the "descriptive model" that is assumed to be implemented in the measurement procedure, and that allows obtaining, as the measurement result, a numerical value of the aimed measurand. Its basic feature, as already stated above, includes the *compensation for the deviations from the ideal state*, in order to bring the result to the reference-state condition, thus obtaining results comparable with those of other measurements on the same measurand, a necessary condition in science. It is more complex, as shown in this article, to obtain it, the *specific descriptive measurement model* (the box corresponds in fig. 7 to fig. 6), whose "output quantities" are equivalent to the "social output quantities" of the prescriptive model.

However, while the latter is most often a purely mathematical one, in which case the "social output quantities" are *computed exact* from it, the "output quantities" of the descriptive model in Fig. 6 are originating from experiments and measurement, and are therefore *affected by uncertainty*, and such is always their comparison with experiments of the same kind, but distinct, of the same or of other authors.

Note: An example of practical application is reported in [8].

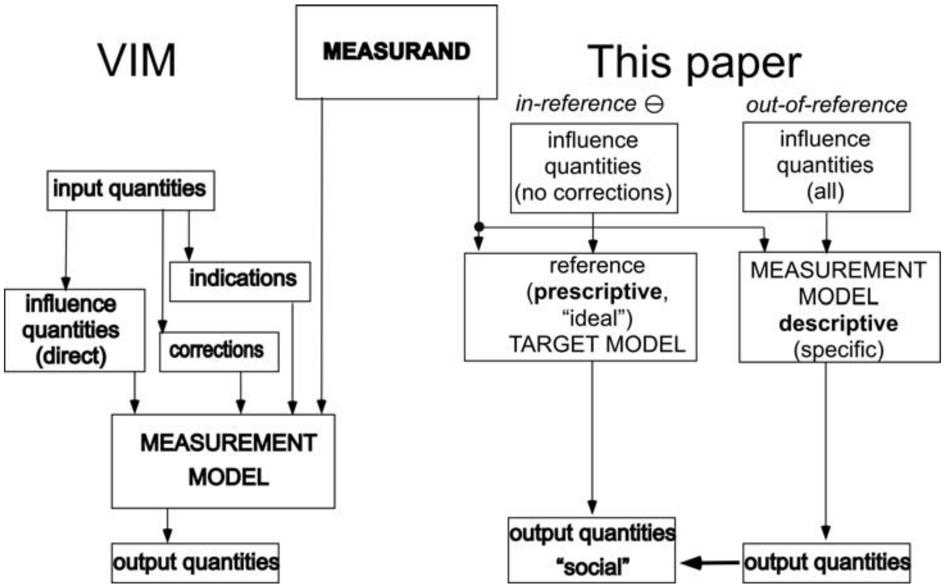

Fig. 7. Comparison between VIM "concept diagram" partially obtained from that of "A.8 measured quantity value" [1] and the concept diagrams proposed in this paper for either the prescriptive (left) and the descriptive (right) models: see text for their desctiption

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МОДЕЛЮВАННЯ В МЕТРОЛОГІЇ: ВІД РОЗПОРЯДЧОЇ МОДЕЛІ ДО ОПИСОВОЇ ДЛЯ ПРАКТИЧНОЇ РЕАЛІЗАЦІЇ ПЕРШОЇ

Ф. Павезе

У науці про вимірюваннях об'єкт вимірювання називається "вимірюваної величиною". Вона визначена в п. 2.3 VIM: 2012 як "величина, яка призначена для вимірювання" (але в п. B.2.9 GUM:1995, вона визначена натомість згідно п. 2.6 VIM:1993, "певна величина, що підлягає вимірюванню". В примітці 1 до п. 2.3 VIM:2012 сказано: "специфікація вимірюваної величини вимагає ... опису [тобто моделі] стану явища, тіла або речовини". Концепція вимірюваної величини повинна бути спільно використовуваною відповідним Співтовариством, оскільки передбачається, що одна і та ж вимірювана величина є об'єктом заходів, що повторюються, які повинні бути такими, що співставляються, тобто вона повинна бути визнана як величина, що має поточне впізнаваєме значення для співтовариства. На діалекті філософів науки це означає, що вона повинна бути спроектована в "соціальні рамки". Також у науковій структурі це означає, модель вимірювання повинна бути "розпорядчого" типу, що означає "давати вказівки або розпорядження" і не завжди означає "фізичну модель". Проект експерименту повинен починатися з цієї вихідної концептуальної моделі вимірюваної величини, "соці-ально розподіленої", а не з побудови описової моделі (часто званої "експериментальною моделлю") вимірювальної системи, яка є специфічною до кожного вимірювального пристрою. Крім того, кожна вимірювальна система повинна бути змодельована на основі конкретних рішень, які обрані для реалізації розпорядження в кожному експерименті. Вихідна концептуальна модель, незалежна від будь-якої конкретної експериментальної реалізації, явно є ідеалізованою. Вона навіть не дозволяє оцінити експериментальні труднощі і компроміс (які поділяються в залежності від цільової невизначеності). Вони виникають з трьох категорій джерел, зазначених в VIM:2012: 1) явище, тіло або речовина; 2) вимірювальна система; 3) умови, при яких проводиться вимірювання. Відповідна модель повинна описувати умови вимірювання (часто звані "фізичною", "експериментальною" або "спостережною" моделлю – тут вона не обов'язково відповідає який-небудь з них). У статті обговорюється непроста схема переходу від розпорядчої моделі до описової.

Ключові слова: *вимірювана величина, модель вимірюваної величини, розпорядча модель, описова модель, умови вимірювання.*

МОДЕЛИРОВНИЕ В МЕТРОЛОГИИ: ОТ ПРЕДПИСЫВАЮЩЕЙ МОДЕЛИ К ОПИСАТЕЛЬНОЙ ДЛЯ ПРАКТИЧЕСКОЙ РЕАЛИЗАЦИИ ПЕРВОЙ

Ф. Павезе

В науке об измерениях объект измерения называется "измеряемой величиной". Она определена в п. 2.3 VIM:2012 как "величина, предназначенная для измерения" (но в п. B.2.9 GUM1995, она определена вместо этого согласно п. 2.6 VIM1993, "определенная величина, подлежащая измерению". В примечании 1 к п. 2.3 VIM:2012 сказано: "спецификация измеряемой величины требует… описания [т.е. модели] состояния явления, тела или вещества". Концепция измеряемой величины должна быть совместно используемой соответствующим Сообществом, поскольку предполагается, что одна и та же измеряемая величина является объектом повторяющихся мер, которые должны быть сопоставимы, то есть она должна быть признана как величина, имеющая текущее узнаваемое значение для сообщества. На диалекте философов науки это означает, что она должна быть спроецирована в "социальные рамки". Также в научной структуре это означает, что измеряемая модель должна быть "предписывающего" типа, что означает "давать указания или предписания" и не всегда означает "физическую модель". Проект эксперимента должен начинаться с этой исходной концептуальной модели измеряемой величины, "социально распределенной", а не с построения описательной модели (часто называемой "экспериментальной моделью") измерительной системы, которая является специфической к каждому измерительному устройству. Кроме того, каждая измерительная система должна быть смоделирована на основе конкретных решений, которые выбраны для реализации предписания в каждом эксперименте. Исходная концептуальная модель, независимая от какой-либо конкретной экспериментальной реализации, явно является идеализированной. Она даже не позволяет оценить экспериментальные трудности и компромисс (которые разделяются в зависимости от целевой неопределенности). Они возникают из трех категорий источников, указанных в VIM:2012: 1) явление, тело или вещество; 2) измерительная система; 3) условия, при которых проводится измерение. Соответствующая модель должна описывать условия измерения (часто называемые "физической", "экспериментальной" или "наблюдательной" моделью – здесь она не обязательно соответствует какой-нибудь из них). В статье обсуждается непростая схема перехода от предписывающей модели к описательной.

Ключевые слова: *измеряемая величина, модель измеряемой величины, предписывающая модель, описательная модель, условия измерения.*